



UNIVERSITÉ DU QUÉBEC EN OUTAOUAIS
DÉPARTEMENT DES SCIENCES ADMINISTRATIVES

WORKING PAPER NO. 18

Housing, Leverage, and Intergenerational Wealth Inequality

A Dynamic General Equilibrium Theory

Simon-Pierre Boucher

Département des sciences administratives
Université du Québec en Outaouais

simon-pierre.boucher@uqo.ca

Gatineau – Pavillon Alexandre-Taché
283, boulevard Alexandre-Taché
Gatineau, Québec, Canada J9A 1L8

October 2026

Version 1.0

Abstract

This paper asks under what conditions inherited wealth, mortgage leverage, nominal debt, housing supply and down-payment constraints generate persistent owner–renter wealth gaps among households with the same labour-income process, and what an unanticipated inflation changes. A three-stage benchmark with two-period generations yields a closed-form ownership-entry threshold that adds a liquidity component to the down-payment bound, a leverage multiplier whose break-even price growth is independent of the loan-to-value ratio, a closed-form intergenerational tenure persistence that is positive only if the house is net wealth at the end of life, existence and a uniqueness condition for the stationary equilibrium, and the accounting of a price-level surprise under three mortgage contracts. A twelve-period overlapping-generations model with nominal annuity mortgages, gifts and estates, and segmented rental and construction sectors is solved in stationary general equilibrium and along perfect-foresight transitions under an illustrative calibration. Transfers generate a 0.95 income-year wealth gap at mid-life and a rank–rank wealth slope of 0.34 that vanishes without transfers; price adjustment removes 64–66% of the partial-equilibrium ownership response to the down-payment constraint; no multiple equilibria arise under near-fixed supply. Because households are net nominal creditors, a 10% surprise moves the real house price by -0.45% , a fall where the benchmark predicts a rise; the contract ordering, fixed-rate above renewable term above adjustable, is confirmed.

Keywords: housing tenure, mortgage leverage, intergenerational wealth, bequests, down-payment constraint, surprise inflation, overlapping generations

JEL Classification: R21, R31, D31, D91, E31, G51

1. Introduction

Among Canadian families under 35, the median net worth of those who own their home is about ten times that of those who do not, and the gap is of the same order among families aged 55–64 ([Statistics Canada, 2024](#)). Differences in labour income between the two groups are far smaller than this. Housing tenure is, in other words, a sorting device for wealth: it converts a modest difference in initial resources, often a parental transfer at the time of first purchase, into a large and durable difference in balance sheets, through the down payment that gates entry, the leverage that scales the return on the house, and the bequest that carries the result into the next generation. This paper asks under what conditions inherited wealth, mortgage leverage, nominal debt, housing supply and down-payment constraints generate persistent owner–renter wealth inequality among households with the same labour-income process, and what an unanticipated inflation changes in that mechanism.

The question is theoretical and the answer is given in two steps. The first is a benchmark economy in which a household lives three stages, buys or rents once, repays a nominal mortgage, sells, and transfers part of the proceeds to a child who enters the model at that moment. Because the benchmark has closed forms, it delivers nine propositions: an ownership-entry threshold that equals the down-payment bound plus the stage-1 consumption of the marginal buyer (Proposition 1); the signs of its comparative statics, all of them liquidity effects of present-value-neutral parameters (Propositions 2–3); a leverage multiplier net of transaction costs whose break-even price growth is independent of the loan-to-value ratio (Proposition 4); a closed-form intergenerational tenure persistence, the second eigenvalue of the dynasty chain, which is positive only if the house is net wealth at the end of life and which is a half-life rather than a trap (Proposition 5); existence of a stationary equilibrium and a dominance condition for its uniqueness, with the corollary that a fixed stock converts every credit or transfer shift into price and composition (Proposition 6, Corollary 1); price neutrality without segmentation under unit demand (Proposition 7); the full accounting of a price-level surprise under fixed-rate, renewable-term and adjustable-rate contracts, with the exact condition under which an indebted owner gains (Proposition 8); and the general-equilibrium impact of a surprise, whose sign is a statement about the net nominal position of the parents whose children are at the tenure margin (Proposition 9, Corollary 2).

The second step is a quantitative overlapping-generations economy that relaxes the benchmark along the dimensions that matter for the question: twelve five-year periods with housing at every age and on an intensive margin, persistent income risk, nominal constant-payment mortgages amortised over 25 years under three contract types with a loan-to-value and a nominal payment-to-income constraint, an inter-vivos gift at the child’s entry and an estate at the parent’s death, segmented rental and construction sectors with durable stocks, and a lender with a balance sheet. It is solved in

stationary general equilibrium and along perfect-foresight transitions after an unanticipated inflation, with the two housing markets clearing to a stated tolerance, and every result at fixed prices is labelled as such. The calibration is illustrative: no moment is fitted, verified Canadian comparison points are reported next to the model's moments, and no verified estimate of the Canadian housing-supply elasticity was found, so that parameter is swept rather than set.

Three findings are the paper's main results and all three are stated with their equilibrium status. First, transfers generate the equal-income gaps the question asks about and are the whole of their intergenerational persistence in this economy. In the baseline, owners hold 0.95 years of average income more than renters at 40–44, children of owners are 0.54 rather than 0.35 likely to own at 35–39, and the rank–rank slope of net worth between parents and children is 0.34; switching the bequest motive off lowers the house price from 1 to 0.931 and sets the tenure persistence and the rank–rank slope to zero. The persistence is a half-life of 0.25 generations, not a trap: the search for multiple stationary equilibria, including under near-fixed supply, finds a monotone excess-demand function with a single root. Second, prices absorb most of what credit constraints would otherwise do to ownership. Lowering the minimum down payment from 20% to 5% raises ownership among households under 35 from 0.38 to 0.429 at fixed prices, but general equilibrium removes 64% of the aggregate ownership effect through the price and the rent; the long-run supply elasticity matters little for this because the demand shift is small and, in the transitions, because the impact elasticity of a durable stock is the product of the long-run elasticity and the depreciation rate. Third, and contrary to the benchmark's Proposition 9, a 10% unanticipated rise in the price level *lowers* the real house price on impact, by 0.45% under fixed-rate contracts and 1.88% when the surprise persists, because households in this economy are net nominal creditors: their deposits exceed their mortgages, so the surprise lowers aggregate household wealth and with it housing demand. The direct redistribution is as the accounting predicts, owners with a mortgage gain 0.63% of lifetime consumption and renters lose 1.00%, and the contract ordering, fixed-rate at least as good as the renewable term and both better than the adjustable rate, holds, with the renewable term penalised precisely when the surprise is persistent and the real rate responds. General equilibrium absorbs 92% of the partial-equilibrium ownership response to the surprise, and the perfect-foresight transition differs from the static-expectations impact of the benchmark by a tenth of a percentage point.

The contributions are stated relative to the closest prior work and without inflation. The entry threshold with a liquidity component is an incremental extension of [Ortalo-Magné and Rady \(2006\)](#) and [Kaplan et al. \(2020\)](#) to transfers, a rental sector and nominal debt; its general-equilibrium offset is a numerical statement, and the price feedback is shown to be governed by segmentation and the intensive margin, as [Greenwald and Guren \(2025\)](#) would predict. The persistence result rationalises in closed form what [Charles and Hurst \(2002\)](#) and [Daysal et al. \(2023\)](#) document empirically; it is new as theory and is deliberately not a multiplicity result. The inflation results are incremental on the

direct revaluation of [Doepke and Schneider \(2006\)](#) and [Meh and Terajima \(2011\)](#) and on the contract comparison of [Garriga et al. \(2017\)](#), and new in two respects: the sign of the general-equilibrium price response is tied to a density-weighted net nominal position, and the Canadian five-year term is placed between the long fixed-rate and the adjustable contract and shown to lose its advantage when a persistent surprise is met by a real-rate response. The supply-elasticity and contract effects on mobility are comparative statics of a mobility measure, incremental on [Kiyotaki et al. \(2011\)](#) and [Chambers et al. \(2009\)](#), and small in this calibration.

Three results in this paper are negative relative to the hypotheses with which it started, and they are reported as results. Surprise inflation lowers rather than raises the real house price when households are net nominal creditors; price neutrality without segmentation does not carry over to an economy with an intensive housing margin; and the entry threshold is not monotone in the down payment at every productivity state once households can buy later. In each case the responsible benchmark assumption is identified and the proposition is restated under it, rather than the theory being adjusted to fit the simulations or the simulations being discarded.

The paper proceeds as follows. Section 2 reviews the literature by mechanism. Section 3 sets out the benchmark and the quantitative economy. Section 4 states the propositions and the remarks that scope them. Section 5 describes the algorithm, the calibration and the verification. Section 6 reports the baseline, the equal-income paths and the down-payment sweep; Section 7 the inflation and real-rate transitions; Section 8 supply, bequests, policy and welfare; Section 9 sensitivity, limitations and conclusions. Appendix A contains the proofs, Appendix B the numerical details and Appendix C the supplementary exhibits.

2. Related literature

The literature that bears on the question is large but fragmented along five mechanisms that are rarely combined in one general-equilibrium model: the collateral or down-payment constraint that gates entry into ownership; the leveraged return on housing and the resulting divergence of owner and renter balance sheets; the revaluation of nominal mortgage debt under surprise inflation; the supply side that converts demand shocks into prices rather than quantities; and the intergenerational transmission of wealth through transfers. This section reviews each mechanism, then the Canadian institutional setting, and closes with the gaps that the model is built to address. Only references whose metadata were verified are cited.

2.1. Collateral, leverage and the price feedback

The amplification mechanism descends from [Kiyotaki and Moore \(1997\)](#): when durable assets serve as collateral, a shock that raises their price relaxes borrowing limits, which raises demand and the

price again. [Iacoviello \(2005\)](#) transplanted the mechanism into a monetary model with housing collateral and nominal debt, obtaining two results this paper inherits, that collateral constraints amplify demand shocks and that debt deflation transfers wealth from borrowers to lenders. Its limits are equally instructive: two representative households, no tenure choice, no wealth distribution and no inheritance, so that the redistribution it identifies is between two types rather than within a cross-section of households that differ in initial wealth.

The quantitative heterogeneous-household literature supplies the architecture. [Favilukis et al. \(2017\)](#) build an overlapping-generations economy with aggregate and idiosyncratic risk, collateral constraints and heterogeneous bequest motives, and show that relaxing financing constraints produces a large price boom through a fall in the housing risk premium. [Kaplan et al. \(2020\)](#) add a rental market, long-term mortgages and tenure choice and reach the conclusion that disciplines any claim about the collateral–price feedback: in their calibration, shifts in credit conditions move ownership, leverage and foreclosures but not prices, because the rental sector and long-term mortgages absorb the demand shift. [Greenwald and Guren \(2025\)](#) resolve the apparent conflict with the credit-driven-boom literature by showing that everything hinges on the segmentation between the owner-occupied and rental stocks. [Sommer et al. \(2013\)](#) and [Sommer and Sullivan \(2018\)](#) provide the template for a rental equilibrium with landlords and an endogenous rent–price ratio; the latter is a standing warning that general-equilibrium price responses can reverse partial-equilibrium intuition. For the present paper this literature means that the strength of any price feedback is not a free parameter: it is governed by the segmentation parameter and the supply elasticity, both of which the model makes explicit, and Proposition 7 restates the [Kaplan et al.](#) neutrality in the model’s language while Remark 2 records the condition, unit demand, under which it holds.

The empirical side of the collateral channel is well established. [Mian and Sufi \(2011\)](#) document home-equity borrowing of 25–30 cents per dollar of house-price gain, concentrated among constrained owners; [Gete and Reher \(2018\)](#) show that tighter mortgage supply pushes households into renting and raises rents; [Cloyne et al. \(2020\)](#) show that the consumption response to interest rates is driven by mortgagors, with outright owners not responding. None of these speak to intergenerational persistence, but together they justify two modelling choices: owners must be allowed to be liquidity-constrained despite high net worth, and the rental market must be an explicit margin rather than an outside option.

The mortgage-contract literature is the second pillar. [Campbell and Cocco \(2003\)](#) formalise the household’s choice between a nominal fixed-rate mortgage, which carries inflation risk, and an adjustable-rate mortgage, which carries income risk. [Chambers et al. \(2009\)](#) embed contract choice in an overlapping-generations economy with tenure choice and attribute a large share of the rise in US ownership to mortgage innovation: contract design changes who can own, which is the sense in which the present paper treats the contract. [Guren et al. \(2021\)](#) and [Campbell et al. \(2021\)](#) study

contract design for macroeconomic stabilisation in equilibrium life-cycle models; the latter prices mortgages off a risk-averse lender, a device that keeps lender wealth in the accounting of inflation redistribution, as the model here does with a simpler risk-neutral lender.

2.2. Nominal debt and surprise inflation

The Fisher channel ([Fisher, 1933](#)) is the oldest element of the paper. Its modern quantification is [Doepke and Schneider \(2006\)](#), who revalue the nominal positions of US households by age and wealth and find that a moderate inflation episode transfers wealth from rich, old bondholders and from foreigners to young middle-class households with fixed-rate mortgages. [Adam and Zhu \(2016\)](#) replicate the exercise for the euro area and [Meh and Terajima \(2011\)](#) for Canada, finding that the main winners are young middle-class households with mortgage debt while the old, the rich and holders of non-indexed pensions lose. [Pallotti et al. \(2024\)](#) bring the accounting up to the 2021–2023 episode and add a nuance that matters for a model with renters: rigid rents acted as a hedge for lower-consumption households, so the episode worked like an age-dependent tax.

These are accounting exercises. They hold prices, rents, interest rates and behaviour fixed and therefore cannot answer whether the direct revaluation survives once rents, house prices and lender wealth respond. Three papers go further. [Meh et al. \(2010\)](#) compute the general-equilibrium response to an unexpected price-level increase in an overlapping-generations economy with nominal positions calibrated for Canada and find that the young, the poor and the government gain and that the policy regime matters; the model has no housing market, so the owner–renter dimension is absent. [Auclert \(2019\)](#) provides the sufficient-statistic language of unhedged interest-rate exposure and the Fisher channel but has no tenure structure or intergenerational dynamics. [Garriga et al. \(2017\)](#) is the closest precedent for the inflation results: in a general-equilibrium model with long-term nominal mortgages they show that persistent inflation benefits fixed-rate borrowers over time but hurts adjustable-rate borrowers immediately. Their borrowers and lenders are two representative types; there is no renter priced out when nominal rates rise, no ownership-entry margin and no inheritance. The lesson this paper draws is that the sign of the general-equilibrium effect of a surprise cannot be assumed: [Garriga et al.](#) show that it is negative for adjustable-rate borrowers, [Pallotti et al.](#) that rent rigidity can protect renters, and [Kaplan et al.](#) that the rental sector can absorb demand. Proposition 9 and Corollary 2 therefore state conditions rather than a sign, and Section 7 finds the sign that the brief expected to be positive to be negative.

2.3. Land scarcity, supply elasticity and the rent–price ratio

The case for treating land as a scarce factor is strong. [Knoll et al. \(2017\)](#) show that real house prices in fourteen countries were roughly flat until the mid-twentieth century and then tripled, with

most of the rise attributable to land; [Davis and Heathcote \(2007\)](#) show that the land share of US house value rose and drives price volatility; [Rognlie \(2015\)](#) shows that the post-war rise in the net capital share comes from housing; [Piketty and Zucman \(2014\)](#) document the rise of wealth–income ratios in which housing is the main component. In the cross-section, [Saiz \(2010\)](#) and [Glaeser and Gyourko \(2018\)](#) establish that geography and regulation determine supply elasticities, and [Gyourko et al. \(2013\)](#) show that inelastic supply with rising high-income demand produces persistent price divergence and sorting.

Two theoretical papers connect supply scarcity to the owner–renter distribution. [Kiyotaki et al. \(2011\)](#) build a life-cycle production economy in which land and capital produce real estate and find that when the land share is large, prices react strongly to productivity and world-interest-rate shocks, redistributing between net buyers and net sellers, while financing constraints have limited price effects. [Grossmann et al. \(2021\)](#) model land as a fixed factor in a growing economy and show that surging rents redistribute from renters to landowners. [Borri and Reichlin \(2021\)](#) analyse optimal taxation when appreciation widens the owner–renter gap, and [Favilukis et al. \(2023\)](#) offer the most complete architecture, owners, renters, construction and endogenous wealth inequality, for evaluating affordability policies. What is missing is the interaction of supply elasticity with the other mechanisms: [Kiyotaki et al.](#) have no bequests and no nominal debt, [Grossmann et al.](#) have representative owner and renter types, and neither models the entry threshold. The conjecture that lower supply elasticity raises the price response to any demand shift and therefore both the leveraged return of incumbents and the entry threshold of renters is plausible but, as Section 8 shows, quantitatively weak when the demand shift itself is small and when the stock is durable.

2.4. Tenure choice, the down-payment constraint and the property ladder

The microeconomics of tenure choice begins with [Henderson and Ioannides \(1983\)](#), whose rental externality rationalises an ownership premium, and with [Linneman and Wachter \(1989\)](#), who show that it is the wealth constraint rather than the income constraint that blocks ownership. [Stein \(1995\)](#) shows that down-payment effects generate multiple equilibria and price–volume comovement in a static setting. The paper closest to the threshold result is [Ortalo-Magné and Rady \(2006\)](#): in a life-cycle model with a property ladder and a down-payment constraint, the ability of young households to finance the down payment on a starter home drives prices, and capital gains on starter homes relax the constraints of existing owners. It has no bequests, no nominal debt, no inflation and no supply side, and the distribution of wealth across households of equal income is not an object of analysis.

The quantitative life-cycle literature fills in the behavioural detail: [Gervais \(2002\)](#), [Iacoviello and Pavan \(2013\)](#), [Halket and Vasudev \(2014\)](#), [Díaz and Luengo-Prado \(2010\)](#), [Kaas et al. \(2021\)](#), [Floetotto et al. \(2016\)](#) and [Halket and Pignatti Morano di Custoza \(2015\)](#). [Paz-Pardo \(2024\)](#) is

especially relevant because it identifies a competing explanation for falling young ownership and wealth, riskier and more unequal earnings for later cohorts; the claim that inheritance generates gaps among households with similar labour income must therefore be isolated from the earnings channel, which is why the earnings process is held fixed across inheritance groups in the baseline and varied separately. [Nakajima and Telyukova \(2020\)](#) close the life cycle: retired owners dissave slowly, so housing is the main asset bequeathed, which ties the tenure block to the bequest block. The quantitative model here departs from that finding in one respect that [Section 6](#) flags: with a warm glow over a liquid estate, old households sell their house before death.

2.5. Incomplete markets, bequests and intergenerational wealth

The incomplete-markets tradition ([Samuelson, 1958](#); [Diamond, 1965](#); [Aiyagari, 1994](#); [Huggett, 1996](#)) provides the equilibrium concept. [De Nardi \(2004\)](#) is the canonical bequest specification, voluntary bequests as a luxury good, which the model adopts and extends to housing. [Benhabib et al. \(2011\)](#) and [Gabaix et al. \(2016\)](#) supply analytical benchmarks for persistence, Pareto tails from capital-income risk and the slow convergence of random-growth processes; they concern the tail of the wealth distribution rather than a tenure-based structure, and the half-life statements of [Proposition 5](#) are the model’s counterpart. [Kuhn et al. \(2020\)](#) provide the macro-historical motivation: over 1949–2016 the portfolio composition of US households, leveraged housing for the middle class and equity for the top, makes asset-price movements the main driver of wealth-inequality dynamics.

The empirical literature on intergenerational wealth has converged on three findings the model must respect. Parental transfers at the time of purchase are a first-order determinant of entry: [Engelhardt and Mayer \(1998\)](#) find that about a fifth of US first-time buyers receive transfers, [Guiso and Jappelli \(2002\)](#) that transfers advance first purchase in Italy, [Blickle and Brown \(2019\)](#) that a transfer raises the renter-to-owner transition probability through relaxed constraints rather than wealth effects, and [Charles and Hurst \(2002\)](#) that parental wealth and transfers explain much of the Black–White ownership gap at equal income. Parental housing wealth specifically transmits to children: [Daysal et al. \(2023\)](#) estimate on Danish registers that about a quarter of a parental housing-wealth gain in early childhood passes through to the child’s adult housing wealth, with earnings and education explaining only a fraction; [Benetton et al. \(2022\)](#) study the US parent–child link through home equity. And transfers are not the whole story: [Fagereng et al. \(2021\)](#) show with adoptees that family environment raises child wealth causally and not mainly through gifts, [Nekoei and Seim \(2023\)](#) show that inheritances reduce inequality on receipt but raise it within a decade because wealthy heirs earn higher returns, and [Boar \(2021\)](#) show that parents save to insure their children’s income risk. The implication is that the inheritance channel formalised here is real and measurable but is not claimed to be the dominant source of intergenerational

wealth persistence; the model isolates one mechanism, transfers interacting with the down-payment constraint and leveraged housing returns, while holding preferences, financial literacy and earnings transmission fixed. The [Nekoei and Seim](#) finding that returns on inherited wealth drive the reversal from equalising to disequalising is exactly what the ownership threshold does: it is a discontinuity in the return to the first dollar of transfer.

2.6. The Canadian institutional setting

Canada differs from the United States in ways that matter for every mechanism above. Mortgages are amortised over 25 years but renewed every five years or less at the prevailing rate, so the fixed-rate contract is fixed only for the term: [Kartashova and Zhou \(2020\)](#) exploit scheduled resets to show asymmetric spending and deleveraging responses; [Bounajm and McWhirter \(2024\)](#) document how the Bank of Canada’s projection model represents the term structure of mortgage debt; [Godbout et al. \(2025\)](#) estimate that a majority of outstanding mortgages renew in 2025–2026 at higher payments; and [Canada Mortgage and Housing Corporation \(2026\)](#) show that traditional five-year fixed contracts were a small minority of new extensions in early 2026. The consequence for the inflation results is that the Canadian contract is intermediate between the US 30-year fixed-rate mortgage of [Doepke and Schneider](#) and the adjustable contract of [Garriga et al.](#): the inflation gain accrues only for the remaining term and is partly clawed back at renewal unless the surprise is transitory. [Allen et al. \(2019\)](#) document search frictions and rate dispersion in the insured segment, which are noted but not modelled. On the household side, [Meh and Terajima \(2011\)](#) provide nominal-exposure profiles by age and tenure, [Hou \(2010\)](#) the age profile of ownership, [Mirdamadi and Khalid \(2023\)](#) the parental-ownership channel and the share of first-time buyers receiving family gifts, and [Statistics Canada \(2024\)](#) the owner–renter wealth gaps quoted in the introduction. No Canadian analogue of the [Saiz](#) elasticity estimates was found among verified sources, so supply elasticity is treated as an illustrative parameter with a range.

2.7. Gaps

Reading the five strands together, four gaps are not filled by any verified paper. No general-equilibrium model combines bequest heterogeneity, a down-payment-gated ownership margin, an explicit rental sector and price feedback, and characterises the entry threshold analytically: [Ortalo-Magné and Rady](#) have the threshold and the feedback but not bequests or a rental sector, [Favilukis et al.](#) have bequest heterogeneity and feedback but no tenure margin, [Kaplan et al.](#) have everything except inheritance heterogeneity and do not characterise the threshold. Persistence of the ownership gap across generations is documented empirically but not derived; the theoretical persistence results that exist concern the tail of the wealth distribution. Surprise inflation with fixed

nominal mortgages has been studied in general equilibrium but never with renters, an entry margin and inheritance, and never with the Canadian renewable term placed between the long fixed-rate and the adjustable contract. And supply elasticity has been linked to owner–renter redistribution and contracts to ownership rates, but neither to intergenerational mobility in tenure or wealth. The model of Section 3 is built to fill these gaps in a form that can be proved where possible and simulated where not; the verdicts on which results are new and which are incremental are given in the introduction and are not revisited in the conclusion.

3. The model

The analysis uses two economies that share notation and mechanisms. The *benchmark* (Section 3.2) compresses the life cycle into three stages and a generation into two periods; it delivers closed forms and the propositions of Section 4. The *quantitative* economy (Section 3.3) has twelve five-year periods, housing on an intensive margin, persistent income risk, long-term nominal annuity mortgages, two bequest dates, durable housing stocks and an explicit lender; it is solved numerically in Sections 5–9. Every reduction made in the benchmark is a candidate explanation for a discrepancy between the propositions and the simulations, and Section 4 names those reductions explicitly.

3.1. Units and timing conventions

Time is discrete. All quantities labelled “real” are in units of the period consumption good. The consumer price index is P_t , gross inflation $1 + \pi_t = P_t/P_{t-1}$, expected inflation over $(t-1, t)$ as of $t-1$ is π_t^e , and the surprise is defined multiplicatively, $1 + \pi_t^u \equiv (1 + \pi_t)/(1 + \pi_t^e)$; in a stationary equilibrium $\pi_t = \pi_t^e = \bar{\pi}$. The real price of one unit of owner-occupied housing is p_t , so the house-price index $P_t p_t$ is distinct from the CPI. The real rent per unit of rental services is ρ_t . A mortgage originated at t has nominal principal B_t^{nom} ; its real value at a later date $t+k$ is $b_{t+k}^m \equiv B_{t+k}^{nom}/P_{t+k}$, so the nominal balance is what the contract fixes and the real balance is what matters for consumption. The real gross rate is $R \equiv 1 + r$, exogenous (a small open economy), and the nominal rate on contracts written at t satisfies the Fisher relation $1 + i_t = R(1 + \pi_{t+1}^e)$ because the lender is competitive and risk neutral. A fixed-rate mortgage (FRM) keeps i_t for the life of the loan; an adjustable-rate mortgage (ARM) resets every period; a term- k contract resets every k periods, which in the five-year calibration of the quantitative model represents the Canadian renewable term. Households belong to dynasties, infinite sequences of parent–child pairs; the labour-income process is identical across dynasties, so dynasties differ only through the transfers they receive. This “equal-income” design is what makes the owner–renter gaps of the paper attributable to transfers, leverage and constraints rather than to earnings.

3.2. The benchmark economy

Demographics and timing. A household lives three stages, $j = 1, 2, 3$: a *buyer* (young adult, who chooses tenure, buys or rents, and borrows), a *payer* (prime age, who pays rent or mortgage interest and amortisation; under an ARM or term contract the rate is reset at the start of this stage) and a *seller–donor* (who sells the house, repays the remaining balance, consumes and gives an inter-vivos transfer to the child). The child enters the model at stage 1 exactly when the parent enters stage 3, so a generation lasts two periods and the transfer arrives at the moment of the tenure decision, the “gift at first purchase” channel documented by [Engelhardt and Mayer \(1998\)](#) and [Blickle and Brown \(2019\)](#). A unit mass of stage-1 households is born each period. Stage-3 households do not demand housing in the two markets (their retirement housing is outside the model and netted out of y_3); this is the one demographic simplification of the benchmark, and the quantitative model removes it. Within a period: the price level and the stage-3 income shock are realised; real prices (p_t, ρ_t) are determined; stage-3 households sell, repay, consume and transfer; stage-1 households receive the transfer and choose tenure, housing and the mortgage while stage-2 households pay and consume; the owner-occupied and rental markets clear.

Preferences and the housing need. A household born at t with tenure $o \in \{O, R\}$ has lifetime utility

$$U^o = \ln c_1 + \chi \mathbf{1}\{o = O\} + \beta \mathbb{E}[\ln c_2] + \beta^2 \mathbb{E}[\ln c_3 + v \ln b'], \quad \beta \in (0, 1), v > 0, \chi \geq 0, \quad (1)$$

where b' is the transfer to the child. Each household needs exactly one unit of housing services at stages 1 and 2 (unit demand): an owner holds one unit of the owner-occupied stock, a renter rents one unit of the rental stock. Housing therefore enters utility only through the constant ownership premium χ ([Henderson and Ioannides, 1983](#)); $\chi = 0$ is allowed. The warm-glow term is the [De Nardi \(2004\)](#) specification with the luxury shift set to zero so that the transfer rule is linear. Incomes $y_1, y_2 > 0$ are deterministic and common; y_3 is drawn from a distribution G with a continuous density $g > 0$ on $[\underline{y}_3, \bar{y}_3]$, independently across households and generations.

Mortgage, deposits and the lender. A stage-1 owner buys one unit at p_t , pays the buying cost $\kappa_b p_t$ and maintenance $m p_t$, makes a down payment of at least θp_t and borrows the nominal principal $B_t^{nom} \leq P_t(1 - \theta)p_t$. The contract pays interest $i_1 B_t^{nom}$ plus amortisation $a B_t^{nom}$ at stage 2, $a \in (0, 1]$, and $(1 + i_2)(1 - a)B_t^{nom}$ at stage 3, with $i_1 = R(1 + \pi_{t+1}^e) - 1$ fixed at origination for every contract and $i_2 = i_1$ (FRM) or $i_2 = R(1 + \pi_{t+2|t+1}^e) - 1$ (ARM or term contract renewed at $t + 1$). Real payments are $D_2 = (i_1 + a)(1 - \theta)p_t / (1 + \pi_{t+1})$ and $D_3 = (1 + i_2)(1 - a)(1 - \theta)p_t / [(1 + \pi_{t+1})(1 +$

$\pi_{t+2})]$. In the stationary state write $D_2 = \delta_2 p$, $D_3 = \delta_3 p$ with

$$\delta_2 \equiv (1 - \theta) \left[R - \frac{1 - a}{1 + \bar{\pi}} \right], \quad \delta_3 \equiv \frac{(1 - a)(1 - \theta)R}{1 + \bar{\pi}}, \quad e_3 \equiv (1 - \kappa_s) - \delta_3, \quad d \equiv \theta + \kappa_b + m, \quad (2)$$

so that d is the stage-1 cash outlay per unit of price and e_3 the net equity share at sale. The identity

$$\delta_2 + \frac{\delta_3}{R} = (1 - \theta)R \quad (3)$$

is the present-value (Fisher) neutrality of the nominal contract: anticipated inflation front-loads real payments ($\partial \delta_2 / \partial \bar{\pi} > 0$, $\partial \delta_3 / \partial \bar{\pi} < 0$), the “tilt”, without changing their present value. The house is net wealth at the end of life, $e_3 > 0$, if and only if $(1 - \kappa_s)(1 + \bar{\pi}) > (1 - a)(1 - \theta)R$, that is, if amortisation and the down payment are large enough; a two-period version in which the whole mortgage is a balloon fails this condition for generation-length periods, which is why the benchmark has three stages. The liquid asset is a one-period nominal deposit $n \geq 0$ paying i_t ; its real gross return is $R/(1 + \pi_{t+1}^u)$, so a surprise erodes nominal savings exactly as it erodes nominal debt. The lender is a competitive risk-neutral intermediary that funds mortgages and takes deposits at the same nominal rate; its realised real profit in a surprise period accrues to rentiers outside the dynasties, and is the mirror image of the households’ revaluation gains.

Rental and owner-occupied supply. Landlords are competitive and risk neutral and the rental stock is segmented from the owner-occupied stock. Per cohort, owner-occupied supply is $S(p) = \bar{H}p^\varepsilon$, $\varepsilon \in [0, \infty)$, with $\varepsilon = 0$ a fixed stock, and rental supply is $S_r(\rho) = \bar{H}_r \rho^{\varepsilon_r}$, $\varepsilon_r \in (0, \infty]$. In the benchmark S is a per-period supply curve (a reversible land-conversion technology); the quantitative model uses the durable stock-flow version whose impact elasticity is $\varepsilon \delta_H$. Nominal wages and rents are indexed to a surprise with elasticities $\eta_w, \eta_\rho \in [0, 1]$ (full indexation in the baseline). The no-segmentation alternative, one stock converted costlessly by landlords, is the [Kaplan et al. \(2020\)](#) and [Greenwald and Guren \(2025\)](#) benchmark and is analysed in Proposition 7.

Budgets. For an owner born at t , with n_1 and n_2 the deposits carried out of stages 1 and 2,

$$c_1 + d p_t + n_1 = b + y_1 + [(1 - \theta)p_t - B_t], \quad (4)$$

$$c_2 + D_2 + m p_t + n_2 = y_2 + n_1 \frac{R}{1 + \pi_{t+1}^u}, \quad (5)$$

$$c_3 + b' = W_3^O \equiv y_3 + (1 - \kappa_s) p_{t+2} - D_3 + n_2 \frac{R}{1 + \pi_{t+2}^u}, \quad (6)$$

where the bracket in (4) is non-negative and zero when the loan-to-value constraint binds. For a renter, $c_1 + \rho_t + n_1 = b + y_1$, $c_2 + \rho_{t+1} + n_2 = y_2 + n_1 R / (1 + \pi_{t+1}^u)$ and $c_3 + b' = W_3^R \equiv y_3 +$

$n_2 R / (1 + \pi_{t+2}^u)$. Deposits cannot be negative, there is no refinancing, and no default: every stage-2 payment can be met out of income and every owner can repay the stage-3 balance out of income plus the sale proceeds (Assumption 3 below). Consumption of a hand-to-mouth household, which sets $n_1 = n_2 = 0$ and borrows the maximum, is written $c_1^O(b) = b + y_1 - dp$, $c_1^R(b) = b + y_1 - \rho$, $c_2^O = y_2 - \delta_2 p - mp$, $c_2^R = y_2 - \rho$, $W_3^O(y_3) = y_3 + e_3 p$ and $W_3^R(y_3) = y_3$.

3.3. The quantitative economy

The quantitative economy keeps the dynasty structure, the nominal contracts and the segmented markets of the benchmark and relaxes everything else. Figure 1 summarises its timing and Table C1 in Appendix C collects the notation.

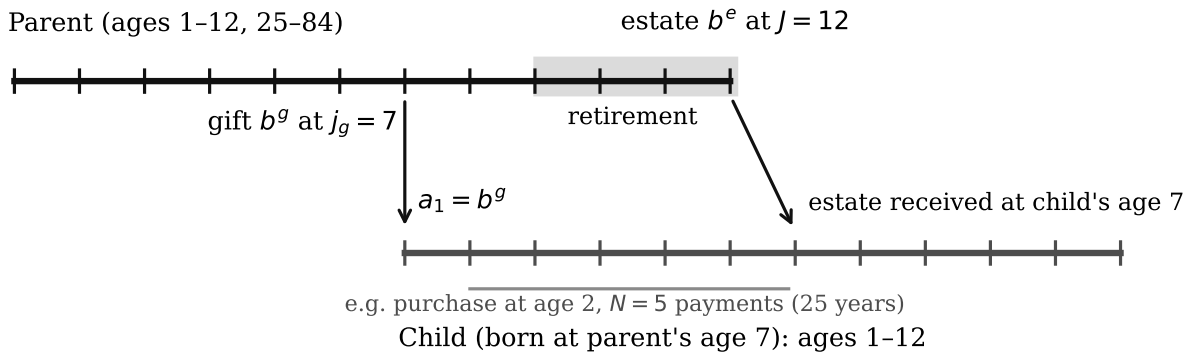


Figure 1. Timing of the overlapping-generations model. One period is five years; a household lives twelve periods (ages 25–84) and retires at 65; its child enters when the parent is 55–59, receives the inter-vivos gift b^g at entry and the estate b^e at the parent’s death, when the child is itself 55–59. A mortgage originated at age 2 is amortised over $N = 5$ periods.

Demographics and transfers. Ages are $j = 1, \dots, J = 12$ (periods of five years, 25–84), retirement at $j_R = 9$. Each household has one child who enters the model at the parent’s age $j_g = 7$; the child receives an inter-vivos gift b^g at its age 1, chosen by the parent at j_g , and the parent’s estate b^e at the parent’s death, when the child is $j_e = 7$. Both are chosen with warm-glow utility; “bequests off” sets the warm-glow weight to zero for both. Population is constant with a unit mass per age, and there is no survival risk in the baseline.

Preferences and income. Period utility over non-housing consumption and housing services is

$$u(c, s) = \frac{(c^{1-\alpha} s^\alpha)^{1-\gamma}}{1-\gamma}, \quad \phi(x) = v \frac{(x + \underline{x})^{1-\gamma}}{1-\gamma}, \quad (7)$$

with $s = (1 + \chi)h$ for an owner of size h and s the rented size for a renter, and ϕ the warm glow over a transfer $x \in \{b^g, b^e\}$ with luxury shift $\underline{x} > 0$ (De Nardi, 2004); the benchmark's linear rule is the limit $\underline{x} \rightarrow 0$, $\gamma \rightarrow 1$. Real labour income at $j < j_R$ is $y_j = \bar{y}_j z$, with $\bar{y}_j$ a hump-shaped age profile and z a five-state Rouwenhorst chain for the five-year aggregate of an annual AR(1); the pension is a flat replacement of the last working-age income. The child's initial state z_1 is drawn from the stationary distribution independently of the parent in the baseline (equal-income design); an intergenerational earnings correlation ρ_g is a sensitivity switch. A small unanticipated transitory component of income, realised before decisions, spreads the atoms of the cash-on-hand distribution that would otherwise make aggregate demand discontinuous in prices (Section 5).

Housing, tenure and transactions. Owner-occupied houses come in four sizes $h \in \mathcal{H}$, the smallest being the indivisibility that creates an entry threshold; rented services $s \geq \underline{s}$ are continuous with $\underline{s} < \underline{h}$. Maintenance costs $m p h$ per period; buying and selling cost $\kappa_b p h$ and $\kappa_s p h$; moving between owned sizes is a sale plus a purchase; renters move costlessly. Each period an owner chooses among *stay* (possibly refinancing at cost $\kappa_f p h$), *move* and *sell-and-rent*; a renter chooses between *rent* and *buy*.

Nominal annuity mortgages and origination constraints. A mortgage with real principal B , nominal contract rate i_0 and $N = 5$ remaining payments (25 years) has the constant nominal payment $\text{pay}^{\text{nom}} = B^{\text{nom}} i_0 / [1 - (1 + i_0)^{-N}]$, and the real balance evolves as

$$b^{m'} = \frac{b^m (1 + i_c) - \text{pay}_t}{1 + \pi_{t+1}}, \quad \text{pay}_t = \text{pay}_t^{\text{nom}} / P_t, \quad (8)$$

where $i_c = i_0$ for an FRM, i_c is the current Fisher rate for an ARM (with the nominal payment recomputed on the remaining balance and term), and i_c resets at every period boundary for the five-year term contract. The real payment of an FRM falls with realised inflation; that of an ARM is re-priced. At origination and refinancing the loan must satisfy the loan-to-value constraint $B \leq (1 - \theta) p_t h$ and a nominal payment-to-income constraint at a qualifying rate $i_q = i_t + \Delta_q$ (the stress test): $\text{pay}^{\text{nom}}(i_q) / (P_t y) \leq \psi$. Because the qualifying payment is nominal and income is current, expected inflation raises i_t and tightens this constraint, which is how the benchmark's tilt reappears in the quantitative model. There is no unsecured borrowing, $a' \geq 0$, and no default; the solver enforces a consumption floor and reports the (zero) mass of forced sales.

Rental sector, construction and the lender. Rental units form a separate stock owned by competitive risk-neutral landlords who buy at p_t^r , pay maintenance and an operating wedge ϕp_t^r , and charge the zero-profit rent

$$\rho_t = p_t^r(1 + m + \phi) - \mathbb{E}_t p_{t+1}^r / R. \quad (9)$$

A segmentation parameter $\sigma \in [0, 1]$ gives the share of rental units that are structurally distinct; the remainder converts one-for-one from the owner-occupied stock at $p_t^r = p_t$, so that $\sigma = 1$ is the benchmark and $\sigma = 0$ the single-stock economy of Proposition 7. Rent rigidity η_ρ applies to sitting tenants in a surprise period, the difference being a transfer from landlords. The two stocks are durable,

$$H_{t+1} = (1 - \delta_H)H_t + \iota p_t^\varepsilon, \quad H_{t+1}^r = (1 - \delta_H)H_t^r + \iota_r (p_t^r)^{\varepsilon_r}, \quad (10)$$

so the stationary supply curves are $S(p) = \iota p^\varepsilon / \delta_H$ and $S_r(p^r) = \iota_r (p^r)^{\varepsilon_r} / \delta_H$ and the one-period elasticity is $\varepsilon \delta_H$. Construction profits go to a class outside the dynasties. The lender holds mortgage balances against household deposits and rentier funding at the world real rate; its real net worth obeys $A_{t+1}^L = (1 + i_t)(\text{Mortgages}_t - \text{Deposits}_t) / (1 + \pi_{t+1}) + \text{rentier flow}$, so a surprise revalues $-(\text{Mortgages}_t - \text{Deposits}_t)(1 + i_t)\pi_{t+1}^u / [(1 + \pi_{t+1}^e)(1 + \pi_{t+1}^u)]$ for the lender, the mirror image of the households' aggregate real gain. A transfer tax τ_b on gifts and estates, rebated lump-sum to entering households, is the policy experiment of Section 8.

The household problem. The individual state at age j is $x = (j, z, o, h, b^m, n_m, a)$ with o tenure, h house size (zero for renters), b^m the real mortgage balance, n_m the remaining payments and $a \geq 0$ the real deposit; along a transition the origination date, hence the FRM rate, is identified by n_m and calendar time. Writing $R_a \equiv (1 + i_{t-1}) / (1 + \pi_t)$ for the realised real deposit return and ℓ for the discrete branch,

$$V_j(x) = \max_{\ell \in \mathcal{L}(x)} V_j^\ell(x), \quad V_j^\ell(x) = \max_{c, a', (h', B', s)} \left\{ u(c, s) + \mathbf{1}_{j=j_g} \phi(b^g) + \beta \mathbb{E}[V_{j+1}(x') | z] \right\}, \quad (11)$$

subject to the branch budget. A renter who rents has $c + \rho s + a' = y_j + aR_a - \mathbf{1}_{j=j_g} b^g$; a renter who buys has $c + (1 + \kappa_b + m)ph' - B' + a' = y_j + aR_a - \mathbf{1}_{j=j_g} b^g$ with the origination constraints; an owner who stays has $c + \text{pay}(b^m, n_m, i_c) + mph + a' = y_j + aR_a - \mathbf{1}_{j=j_g} b^g$; an owner who moves sells at $(1 - \kappa_s)ph$, repays $b^m(1 + i_c)$ and proceeds as a buyer; an owner who sells and rents has $c + \rho s + a' = y_j + aR_a + (1 - \kappa_s)ph - b^m(1 + i_c) - \mathbf{1}_{j=j_g} b^g$. At J the house is sold, the balance repaid and $V_{J+1} \equiv \phi(b^e)$ with b^e the terminal wealth. The estate received at j_e is anticipated as a lottery whose points are equilibrium objects (Section 5). The solver works in real variables; inflation enters only through R_a and the real FRM payment, through the nominal rate in the qualifying payment and in ARM repricing, and through the indexation parameters. Net worth is $a + \mathbf{1}_O[(1 - \kappa_s)ph - b^m]$.

4. Equilibrium and propositions

This section states the analytical results of the benchmark economy. Proofs are in Appendix A. Each proposition is followed by the assumption that the quantitative economy relaxes and, where the simulations of Sections 6–8 contradict the benchmark, by a remark that identifies the responsible assumption. Three results are stated more narrowly than in a first draft of this paper precisely because the simulations contradicted their broader versions: the monotonicity of the entry threshold in the down payment (Proposition 2), price neutrality without segmentation (Proposition 7) and the sign of the house-price response to a surprise (Proposition 9 and Corollary 2). Simulation findings are never called theorems.

4.1. Assumptions

Assumption 1 (Preferences, demography, housing need). Households have the preferences (1), live three stages, need one unit of housing services at stages 1 and 2 (unit demand), and have the income process of Section 3.2: $y_1, y_2 > 0$ common and deterministic, $y_3 \sim G$ with continuous density $g > 0$ on $[\underline{y}_3, \bar{y}_3]$, $0 < \underline{y}_3 < \bar{y}_3 < \infty$, independent across households and generations.

Assumption 2 (Prices, contracts, lender). The real rate $r \geq 0$ is exogenous and $1 + i_t = R(1 + \pi_{t+1}^e)$. A mortgage of real principal $B \leq (1 - \theta)p_t$ is the nominal contract of Section 3.2 with $a \in (0, 1]$; deposits pay i_t and cannot be negative; the lender is competitive and risk neutral and its realised real profit accrues outside the dynasties; the rental stock is segmented, with $S(p) = \bar{H}p^\varepsilon$, $\varepsilon \in [0, \infty)$, and $S_r(\rho) = \bar{H}_r\rho^{\varepsilon_r}$, $\varepsilon_r \in (0, \infty]$, per cohort.

Assumption 3 (Feasibility and no default). For all prices considered, $c_2^O > 0$, $c_2^R > 0$ and $\underline{y}_3 + \min\{e_3p, 0\} > 0$.

Assumption 4 (Liquidity-constrained at the margin: hand-to-mouth). On the wealth interval $\mathcal{B} = (dp - y_1, \bar{b}]$ relevant for the tenure decision, the strict Euler inequalities

$$\frac{1}{c_1^o(b)} > \beta R \frac{1}{c_2^o}, \quad \frac{1}{c_2^o} > \beta R(1 + v) \mathbb{E} \left[\frac{1}{W_3^o(y_3)} \right], \quad o \in \{O, R\}, \quad b \in \mathcal{B}, \quad (12)$$

hold for both tenures at the corner allocation $n_1 = n_2 = 0$ (and $B = (1 - \theta)p$ for owners).

Assumption 5 (Interior tenure margin). $\Delta > 0$ and $dp > \rho$, where

$$\Delta \equiv \chi + \beta [\ln c_2^O - \ln c_2^R] + \beta^2(1 + v) \mathbb{E} [\ln W_3^O(y_3) - \ln W_3^R(y_3)], \quad \Lambda \equiv e^{-\Delta} \in (0, 1). \quad (13)$$

Assumption 6 (Static expectations on impact). In the period of an unanticipated surprise, stage-1 households evaluate their stage-2 rent and stage-3 resale price at the pre-shock stationary values (ρ^*, p^*) while transacting at the current prices.

Assumption 7 (Nominal positions at the shock date). At the date of a surprise the only nominal claims held by households are mortgage balances: stage-3 parents and stage-1 entrants hold no deposits.

Assumption 4 is verified, not imposed, in the numerical benchmark: the Euler slacks are computed on the support of the stationary transfer distribution and a general optimiser returns the corner (`tests/test_toy_model.py`). Assumption 4 implies Assumption 7 on the hand-to-mouth region, but the two are stated separately because the sign of the inflation result depends on the second and not on the first (Corollary 2). Δ collects everything that happens after the purchase, the stage-2 cash-flow comparison and the stage-3 equity; the tenure decision compares the stage-1 cash sacrifice $dp - \rho$ with Δ .

4.2. The ownership-entry threshold

Proposition 1 (Ownership threshold). *Under Assumptions 1–5 there is a unique $\hat{b} \in \mathcal{B}$ such that a household with transfer $b \in \mathcal{B}$ owns if and only if $b \geq \hat{b}$, where*

$$\hat{b} = \frac{dp - \Lambda\rho}{1 - \Lambda} - y_1, \quad \hat{b} - (dp - y_1) = \frac{\Lambda(dp - \rho)}{1 - \Lambda} > 0, \quad c_1^O(\hat{b}) = \frac{\Lambda(dp - \rho)}{1 - \Lambda}. \quad (14)$$

If instead $dp \leq \rho$, every household with $b > dp - y_1$ owns and the threshold is the feasibility bound $dp - y_1$.

The threshold is the sum of a feasibility component, the down payment and costs net of stage-1 income, and a preference–liquidity component equal to the stage-1 consumption of the marginal buyer. The latter depends on the rent, the costs, the amortisation schedule, expected inflation and the bequest motive through Λ . The proposition is proved on the hand-to-mouth region; for households rich enough to save, the envelope derivative of each value is still $1/c_1^O$, so $V^O - V^R$ increases wherever the would-be owner consumes less than the would-be renter, but single crossing at every wealth level is not proved (Remark 4 in Appendix A). The quantitative economy finds exactly one sign change of the buy-minus-rent value along cash for every age and productivity state (Section 6).

Proposition 2 (Comparative statics of the threshold). *Under Assumptions 1–5, with the rent held fixed, the resale price moving with p (stationary comparative statics), and the loan-to-value*

constraint binding,

$$\begin{aligned} \frac{\partial \hat{b}}{\partial p} > 0, \quad \frac{\partial \hat{b}}{\partial \rho} < 0, \quad \frac{\partial \hat{b}}{\partial \theta} > 0, \quad \frac{\partial \hat{b}}{\partial \kappa_b} > 0, \quad \frac{\partial \hat{b}}{\partial \kappa_s} > 0, \\ \frac{\partial \hat{b}}{\partial m} > 0, \quad \frac{\partial \hat{b}}{\partial \chi} < 0, \quad \frac{\partial \hat{b}}{\partial y_1} = -1, \quad \frac{\partial \hat{b}}{\partial \pi} > 0. \end{aligned}$$

The signs of $\partial \hat{b} / \partial \theta$, $\partial \hat{b} / \partial \kappa_s$, $\partial \hat{b} / \partial m$ and $\partial \hat{b} / \partial \pi$ use the fact that, in the benchmark, a renter never buys later, so that V^R does not depend on these parameters.

All signs are liquidity effects. A higher down payment or a more front-loaded real payment schedule is present-value neutral under (3) but moves resources from the constrained young to the unconstrained old; $r \geq 0$ guarantees that the price effect on the outlay dominates the price effect on future equity.

Remark 1 (Scope of Proposition 2: the option to buy later). The sign of $\partial \hat{b} / \partial \theta$ is proved under the benchmark's structure in which the tenure decision is made once: a renter at stage 1 rents for life, so the value of renting is independent of the mortgage parameters. In the quantitative economy a renter can buy at any later age and the renter's continuation value therefore rises when θ falls, through the option of buying later with a smaller down payment. The entry threshold at a given age is then the crossing of two functions that both move with θ , and its monotonicity is not implied by the proposition. Section 6 finds the threshold at the median productivity state to be non-monotone in θ at fixed prices, within the grid sensitivity of that statistic, and finds it unchanged at the lowest state because the payment-to-income cap, not the loan-to-value cap, binds there; it rises with θ from 0.20 to 0.40 as the proposition predicts. The proposition should therefore be read as a statement about the once-and-for-all tenure decision of a liquidity-constrained household whose loan-to-value constraint binds.

Proposition 3 (Anticipated inflation: tilt versus neutrality). (i) Under Assumptions 1–5, an increase in π raises the real stage-2 payment and lowers the real stage-3 payment of a nominal mortgage, leaves their present value unchanged, and raises $\hat{b}$. (ii) For a household whose optimal plan has $n_1 > 0$ and $n_2 > 0$, a small change in π or in θ leaves its value, consumption and transfer unchanged.

4.3. Leverage with transaction costs

Proposition 4 (Leverage multiplier, break-even growth, underwater threshold). Let an owner buy at p , hold a mortgage $(1 - \theta)p$ and resell at $(1 + g)p$ at stage 3. Under Assumption 2:

1. End-of-life wealth responds to the resale price with $(1 - \kappa_s)$ per unit of price, i.e. $(1 - \kappa_s)/d$ per unit of cash invested at purchase; this multiplier is decreasing in θ , κ_b , κ_s and m .

2. Compared with a renter who follows the same consumption path and invests the cash-flow differences at R , the owner has higher stage-3 wealth if and only if $g \geq g^*$, with

$$1 + g^* = \frac{R^2(1 + \kappa_b + m) + mR - (\rho/p)(R + R^2)}{1 - \kappa_s}, \quad (15)$$

increasing in κ_b , κ_s and m , decreasing in ρ/p , and independent of θ .

3. A price decline of x leaves the owner with no housing equity if and only if $x \geq x^* \equiv 1 - \delta_3/(1 - \kappa_s)$; a renter's wealth is unaffected.

Leverage scales gains and losses but, with borrowing at the deposit rate, does not change the break-even: the brief's "leverage amplification" is a statement about the multiplier and the underwater threshold, not about the mean return. In the stationary state the owner-renter end-of-life wealth gap among equal-income hand-to-mouth households is exactly $e_3 p$ and the transfer gap is $ve_3 p/(1 + v)$ (Proposition 5).

4.4. Dynasties and persistence

Proposition 5 (Tenure transitions, persistence and the equal-income wealth gap). *Under Assumptions 1–5, with stationary prices and the stationary transfer support contained in $\mathcal{B}$, let $\hat{y}_R \equiv (1 + v)\hat{b}/v$ and $\hat{y}_O \equiv \hat{y}_R - e_3 p$. Then:*

1. *The child of a renter owns iff $y_3 \geq \hat{y}_R$ and the child of an owner owns iff $y_3 \geq \hat{y}_O$; the tenure of a dynasty is a Markov chain with $q_{RO} = 1 - G(\hat{y}_R)$ and $q_{OO} = 1 - G(\hat{y}_O)$.*
2. *The second eigenvalue of the chain,*

$$\lambda = q_{OO} - q_{RO} = G(\hat{y}_R) - G(\hat{y}_R - e_3 p), \quad (16)$$

equals the parent-child correlation of the ownership indicator and the per-generation persistence of any difference in ownership rates between groups of dynasties; $\lambda > 0$ iff $e_3 > 0$ and $g > 0$ somewhere on $(\hat{y}_O, \hat{y}_R)$; for fixed $\hat{b}$, λ is non-decreasing in $e_3 p$.

3. *For $\lambda < 1$ the chain has the unique stationary owner share $\mu^* = q_{RO}/(1 - \lambda)$ and converges to it geometrically at rate λ .*
4. *Among dynasties with identical income processes, the expected transfer received by an owner's child exceeds that of a renter's child by $ve_3 p/(1 + v)$, and the end-of-life wealth gap between an owner and a renter with the same y_3 is $e_3 p$.*

Persistence is the probability mass of income draws for which parental housing equity is pivotal for the child's tenure. It is zero if the house is not net wealth at the end of life, and it is a half-life, $\ln 2/(-\ln \lambda)$, not a trap: without income risk the two tenure classes would be absorbing and a

continuum of history-dependent steady states would exist, but any G with positive density between the cutoffs makes the chain ergodic (Remark 5 in Appendix A). The paper therefore does not claim path dependence or multiplicity as a generic result.

4.5. Stationary equilibrium

Definition 1 (Stationary equilibrium of the benchmark). A stationary equilibrium is (p^*, ρ^*, μ^*) with $\hat{b}^* = \hat{b}(p^*, \rho^*)$ such that $\mu^* = q_{RO}/(1 - \lambda)$ evaluated at (p^*, ρ^*) , $S(p^*) = \mu^*$ and $S_r(\rho^*) = 1 - \mu^*$ per cohort, the lender earns zero expected real profit, and the distribution of transfers is the stationary distribution of the dynasty chain.

Write $f(\mu) \equiv \mu^*(S^{-1}(\mu), S_r^{-1}(1 - \mu))$ for the owner share generated by the dynasty chain when both markets clear at a cohort owner share μ , with $\mu^* = 0$ when $\Delta \leq 0$ and $\mu^* = 1$ when $\hat{b} < 0$.

Proposition 6 (Existence; uniqueness under a dominance condition). *Let Assumptions 1–3 hold for all (p, ρ) with $p \in [0, S^{-1}(1)]$, $\rho \in [0, S_r^{-1}(1)]$, with $\varepsilon > 0$ and $\varepsilon_r < \infty$. Then a stationary equilibrium exists. If, in addition, Assumptions 4–5 hold along the clearing curve and*

$$(C) \quad \frac{1 + v}{v} \frac{\partial \hat{b}}{\partial p} \geq e_3 \quad \text{for all } \mu \in (0, 1), \quad (17)$$

then f is non-increasing, strictly decreasing wherever $g(\hat{y}_R) > 0$, and the stationary equilibrium is unique.

Condition (C) says that the entry-threshold channel, a higher price raises $\hat{b}$ and excludes renter-dynasty children, dominates the ladder channel, a higher price raises the equity $e_3 p$ that owner-parents bequeath and includes owner-dynasty children. If (C) fails, multiple stationary equilibria of the Stein (1995) type are possible; they are not established here, and Section 8 searches for them numerically and finds none.

Corollary 1 (Fixed owner-occupied stock). *If $\varepsilon = 0$, every stationary equilibrium has $\mu^* = \bar{H}$, and under (C) the price p^* is the unique solution of $\mu^*(p, S_r^{-1}(1 - \bar{H})) = \bar{H}$. A parameter change that raises μ^* at given prices (a lower θ , lower transaction costs, a higher χ or y_1) raises p^* and leaves the ownership rate unchanged; it changes the composition of owners and, through $e_3 p^*$ and $\hat{b}^*$, the persistence λ .*

Proposition 7 (Price neutrality without segmentation, under unit demand). *Suppose, instead of the segmented stocks of Assumption 2, that owner-occupied and rental units are the same stock, converted costlessly by competitive risk-neutral landlords with maintenance m and operating wedge*

$\phi \geq 0$, and that Assumption 1 (unit demand) holds. Then in any stationary equilibrium p^* solves $S(p^*) = 1$ per cohort and $\rho^* = p^*(1 + m + \phi - 1/R)$: the price and the rent are independent of θ , κ_b , κ_s , χ , v , G , the distribution of transfers and of inflation surprises; tenure shares, $\hat{b}^*$ and λ adjust to these prices without feeding back on them.

Remark 2 (Scope of Proposition 7: the intensive margin). The neutrality is a consequence of unit demand: every household occupies one unit whatever its tenure, so credit, transfers and inflation can only reshuffle who owns, not how many units are demanded. With an intensive margin, house sizes and rented services that respond to wealth and prices, total housing demand depends on the wealth distribution, and a shock that lowers household wealth lowers the single-stock price. Section 7 finds that the price response to a surprise in the unsegmented quantitative economy (-0.39%) is almost as large as in the segmented one (-0.45%): in the quantitative model the price feedback is mostly a wealth effect on total housing demand, not the tenure-composition effect that the benchmark isolates. The Kaplan et al. (2020) neutrality is thus recovered only in its unit-demand form.

4.6. Surprise inflation

Two experiments are considered from a stationary equilibrium. (E1) A *level* surprise at $t + 1$: $1 + \pi_{t+1} = (1 + \bar{\pi})(1 + \pi^u)$ and expectations for $t + 2$ onward unchanged. (E2) A *persistent* surprise: $\pi_{t+1} = \pi_{t+2} = (1 + \bar{\pi})(1 + \pi^u) - 1$, learned at $t + 1$, so that ARMs reset to the higher nominal rate while FRMs do not, with an optional real-rate response $r' = r + \phi_r \pi^u$ for $(t + 1, t + 2)$.

Proposition 8 (Direct revaluation and its incidence). *Holding real prices fixed:*

1. For the cohort aged 2 at $t + 1$, the real stage-2 payment is $\delta_2 p / (1 + \pi^u)$ for every contract. The real stage-3 payment is $\delta_3 p / (1 + \pi^u)$ for every contract under (E1); under (E2) it is $\delta_3 p / (1 + \pi^u)^2$ for an FRM and $\delta_3 p (R' / R) / (1 + \pi^u)$ for an ARM, $R' = 1 + r'$. The FRM borrower gains at least as much as the ARM borrower, strictly under (E2), and the ARM stage-3 payment rises in real terms iff $\phi_r > R$.
2. For the cohort aged 3 at $t + 1$, the real stage-3 payment is $\delta_3 p / (1 + \pi^u)$ for every contract.
3. A household holding nominal deposits n loses $nR\pi^u / (1 + \pi^u)$ in real terms; a household whose nominal wage is indexed with elasticity η_w loses $y[1 - (1 + \pi^u)^{-(1-\eta_w)}]$ of real income in the surprise period; a sitting tenant whose rent is indexed with elasticity η_ρ gains $\rho[1 - (1 + \pi^u)^{-(1-\eta_\rho)}]$ at the landlord's expense.
4. The sum over households of real gains on nominal positions equals the lender's real loss: the revaluation is zero-sum. With Γ_3 the real stage-3 gain of the cohort aged 2 ($\Gamma_3 = \delta_3 p \pi^u / (1 + \pi^u)$ under (E1) for both contracts, $\delta_3 p [1 - (1 + \pi^u)^{-2}]$ for an FRM and $\delta_3 p [1 - (R' / R) / (1 + \pi^u)]$ for an ARM under (E2)), an owner of that cohort with deposits n gains on net over its remaining

life iff

$$\delta_2 p \frac{\pi''}{1 + \pi''} + \Gamma_3 > y_2 \left[1 - (1 + \pi'')^{-(1-\eta_w)} \right] + nR \frac{\pi''}{1 + \pi''}. \quad (18)$$

Four classes of owners lose from a surprise: ARM borrowers when the real rate rises by more than R per unit of surprise, owners with imperfectly indexed wages when the real-income loss exceeds the debt erosion, owners holding nominal deposits larger than their mortgage, and, in general equilibrium, new buyers in inelastic markets (Proposition 9, item 4). The claim that inflation enriches owners is false in general; condition (18) is what replaces it.

Proposition 9 (General-equilibrium impact of a level surprise under pure debtor positions). *Let the economy be in a stationary equilibrium satisfying Assumptions 1–5 and 7 with $g(\hat{y}_O) > 0$, and let a level surprise $\pi'' > 0$ occur at $t + 1$ (any contract). Under Assumption 6, with stage-2 owners not transacting, owner-market clearing $2S(p_{t+1}) = \mu^* + \mu_{t+1}$, rental clearing $2S_r(\rho_{t+1}) = 2 - \mu^* - \mu_{t+1}$, and the static-expectations dominance condition*

$$(C') \quad \frac{1 + v}{v} \frac{\partial \hat{b}}{\partial p} \Big|_{p''=p^*} \geq 1 - \kappa_s, \quad (19)$$

the impact responses satisfy:

1. $dp_{t+1}/d\pi'' \geq 0$ and $d\rho_{t+1}/d\pi'' \leq 0$, both strict if $\varepsilon < \infty$ (respectively $\varepsilon_r < \infty$); if $\varepsilon = 0$ the price response is strictly positive and $\mu_{t+1} = \mu^*$.
2. $d\mu_{t+1}/d\pi'' \geq 0$, strictly if $\varepsilon > 0$.
3. $d\hat{b}_{t+1}/d\pi'' \geq 0$; q_{RO} falls, q_{OO} rises, and λ rises.
4. Stage-3 owner-sellers gain $\delta_3 p^* \pi'' / (1 + \pi'') + (1 - \kappa_s) dp_{t+1} \geq 0$; the lender loses the nominal erosion; the stage-1 cash of an inframarginal owner-dynasty child changes by $\frac{v}{1+v} \left[\delta_3 p^* \frac{\pi''}{1+\pi''} + (1 - \kappa_s) dp_{t+1} \right] - d dp_{t+1}$, which is negative when dp_{t+1} is large relative to the erosion and $v/(1+v) < d/(1 - \kappa_s)$; renter-dynasty children who rent in both situations gain iff $\rho_{t+1} < \rho^*$.
5. Without segmentation (Proposition 7) items 1–3 are zero and only Proposition 8 applies.

The mechanism runs entirely through the parents whose debt is eroded: their larger transfers let more owner-dynasty children qualify, the price rises and the rent falls, the threshold rises, and renter-dynasty children at the margin are displaced. Assumption 7 is what makes the parents' revaluation a gain. The next result drops it.

Corollary 2 (Net nominal creditors reverse the sign). *Replace Assumption 7 by predetermined nominal deposits $n^O \geq 0$ and $n^R \geq 0$ held at the shock date by owner-parents and renter-parents of the cohort aged 3 at $t + 1$, chosen before the surprise, and keep the other assumptions of Proposition 9. Define*

$$\Xi \equiv \mu^* g(\hat{y}_O) (\delta_3 p^* - R n^O) - (1 - \mu^*) g(\hat{y}_R) R n^R. \quad (20)$$

Then $dp_{t+1}/d\pi^u$, $d\mu_{t+1}/d\pi^u$ and $-d\rho_{t+1}/d\pi^u$ have the sign of Ξ , with the same strictness conditions as in Proposition 9; Proposition 9 is the case $n^O = n^R = 0$. If $\Xi < 0$, the real house price and the stage-1 ownership rate fall and the rent rises on impact, and $\hat{b}$ falls; the change in q_{RO} is the sum of a price term, positive when $\Xi < 0$, and a direct deposit-loss term $-g(\hat{y}_R)Rn^R/(1 + \pi^u)^2 \leq 0$, so it is unsigned unless $n^R = 0$, in which case q_{RO} rises and λ falls.

Remark 3 (Scope of Proposition 9: the sign is a statement about net nominal positions). Ξ is a density-weighted net nominal debtor position of the parents whose children are at the tenure margin. In the quantitative economy households are net nominal creditors in aggregate (real deposits 4.12 against mortgage debt 2.47 per unit age mass), so the premise of Proposition 9 fails and Section 7 finds, as the corollary indicates, that the real house price *falls* on impact. Two further differences from the corollary are also traceable to benchmark reductions. The rent falls rather than rises, because with an intensive margin the wealth loss of nominal creditors lowers the total demand for housing services, and the two markets are no longer mirror images of a fixed unit demand. And the entry threshold rises rather than falls, because would-be buyers are themselves nominal creditors whose saving for the down payment is eroded, a channel absent from a benchmark in which entrants hold nothing before the transfer arrives. The proposition also covers only the first period under per-period supply and static expectations; the perfect-foresight transition with durable stocks, the second-round effects through the cohort aged 2, which are contract-dependent, and the welfare of marginal households are numerical objects.

4.7. Equilibrium concepts for the quantitative economy

A *stationary recursive competitive equilibrium* with constant inflation $\bar{\pi}$ and exogenous r consists of value functions and policies solving (11) at prices (p^*, p^{r*}, ρ^*) with $i^* = R(1 + \bar{\pi}) - 1$, stocks (H^*, H^{r*}) and a distribution Φ^* over states such that: the owner-occupied market clears, $\int h \mathbf{1}\{o = O\} d\Phi^* = H^* = \iota(p^*)^\varepsilon / \delta_H$; the rental market clears, $\int s \mathbf{1}\{o = R\} d\Phi^* = H^{r*}$ with (9) and $H^{r*} = \iota_r(p^{r*})^{\varepsilon_r} / \delta_H$ on the segmented margin; the distribution of gifts received at age 1 equals the distribution of gifts chosen at j_g , and likewise for estates at j_e , with the estate lottery used by households equal to the distribution of estates in Φ^* ; Φ^* is invariant under the policies, the income chain and the dynasty link; and the lender earns zero expected real profit. A *transition equilibrium* after an unanticipated shock at $t = 0$ is a path of prices, stocks and distributions such that households optimise with perfect foresight of the post-shock path, markets clear every period, pre-shock FRM cohorts keep their origination rate, and the economy is within tolerance of the stationary equilibrium at a terminal date T . Partial-equilibrium counterparts, which re-solve the household block and the distribution at the stationary prices, are reported separately and are never labelled general equilibrium.

5. Numerical algorithm, calibration and verification

5.1. Solution method

The household problem (11) is solved by backward induction over the twelve ages. The discrete branches other than *stay* depend on the state only through cash on hand, so the values of renting, buying, refinancing and selling are computed on a cash-on-hand grid of 120 points and the value of staying on the full state grid (five productivity states, four house sizes, 20 mortgage balances, five remaining-payment counts, 50 liquid-asset points). The continuous choice of next-period liquid assets is an exhaustive search on the grid refined by a golden-section step; loans are chosen among eleven fractions of the binding constraint, gifts among twelve fractions of cash, and the estate is anticipated as a five-point lottery whose points are the conditional means of the simulated distribution of estates by quintile. Endogenous-grid methods were not used because the combination of discrete tenure and size choices with the two origination constraints and a mortgage grid makes the discrete-choice upper envelope fragile, while the cost of the exhaustive search under just-in-time compilation is low (one evaluation of the household block and the distribution takes about one second).

The distribution is a Monte-Carlo panel of 96,000 dynasty slots, each holding a parent and a child whose life-cycle phases are equidistributed, simulated with common random numbers for 1,152,000 household-periods after a burn-in of ten generations. A panel rather than a histogram is used because the dynastic link, a gift at the child's age 1 and an estate at age 7 both correlated with the parent's state, requires the joint parent-child distribution, which a histogram over individual states does not carry. Two approximations are deliberate: the estate is anticipated as the unconditional lottery, although the realised estate in the panel is exactly linked to the parent; and along a transition the lottery is held at its stationary value. A small unanticipated transitory income component, realised before decisions, is added because entrants without a gift and liquidity-constrained renters otherwise share the same cash at a given productivity state and switch tenure simultaneously, which produced a discontinuity in aggregate rental demand of about 1.5% over a 0.05% price interval in a first run; with the component, excess demand is continuous and the root is bracketed.

Market clearing uses a nested regula-falsi (Illinois) search, the rental price inside the owner-occupied price, followed by a damped fixed point on the estate lottery and on the transfer-tax rebate. A Broyden update on the two prices was tried first and abandoned: the excess-demand functions are smooth but strongly non-linear (tenure shares move by 15–20 percentage points for a 5% change in the relative price) and the Monte-Carlo noise of about 10^{-3} in differences of excess demand for a 0.5% price step corrupted the secant updates. The supply scales $\bar{H}$ and $\bar{H}_r$ are set so that the baseline clears at $p^* = p^{r*} = 1$, the normalisation of [Kaplan et al. \(2020\)](#); every counterfactual keeps these scales. Transitions are solved by shooting in the sequence space on the log price paths over $T = 14$

periods (70 years) with the durable stocks (10), a finite-difference Jacobian computed in parallel, Broyden updates with periodic refresh, and, for large shocks, a damped tâtonnement pre-phase. The tolerances are 5×10^{-4} on relative excess demand in the stationary equilibrium, which pins the equilibrium price to about $\pm 0.05\%$ given the Monte-Carlo noise, and 1.5×10^{-3} per period along a transition, where each cross-section has one sixth of the stationary sample. Every run records its residuals; runs that stall above tolerance are labelled with their residual wherever they appear.

Welfare is measured by the consumption-equivalent variation λ^{CE} such that scaling non-housing consumption by $1 + \lambda^{CE}$ in every period of the baseline makes an age-1 household with state (z_1, a_1) as well off as in the alternative,

$$V_1^{alt}(z_1, a_1) = (1 + \lambda^{CE})^{(1-\alpha)(1-\gamma)} U_1^{base}(z_1, a_1) + \Phi_1^{base}(z_1, a_1), \quad (21)$$

where U is discounted flow utility and Φ the discounted warm glow, the latter obtained from cohort simulations under baseline policies and the former as $U = V - \Phi$, an identity at the baseline; the simulated U is kept as a check and differs from $V - \Phi$ by at most 4.9% because simulated policies use nearest-grid loan and size choices, a discrepancy that would inflate the variation if used directly. For households alive at a shock the same measure is applied to their remaining lifetime. Comparisons across scenarios that change preference parameters are reported but are not welfare statements, since they compare value functions under different utilities.

5.2. Calibration

The calibration is illustrative. Table B1 in Appendix B lists every parameter, its unit and its status; no moment is fitted, and the verified Canadian comparison points of Table 1 are reported next to the model's moments without being targeted. The period is five years. The annual discount factor is 0.96, the curvature $\gamma = 2$, the housing share $\alpha = 0.20$; the warm-glow weight $\nu = 2$ and luxury shift $\underline{x} = 1$ were chosen so that mean gifts are about two years of income. Productivity follows an annual AR(1) with persistence 0.95 and innovation standard deviation 0.15, converted to the five-year period, with a hump-shaped age profile and a 50% replacement rate in retirement. The four house sizes are $\{1, 1.5, 2.25, 3.4\}$ units and rented services are continuous above 0.3 units. Maintenance is 1% of value per year, buying costs 2.5% of the price, selling 5%, refinancing 1%, depreciation of the stocks 1% per year. Mortgages are amortised over $N = 5$ periods, the Canadian 25-year standard; the baseline minimum down payment is $\theta = 0.20$, the uninsured threshold; the payment-to-income cap is $\psi = 0.39$ at a qualifying rate two percentage points above the contract rate. The real rate and trend inflation are 2% per year, the latter the Bank of Canada target, so the five-year nominal rate is 0.219. The landlord wedge is 5% of value per period, segmentation is full and both supply elasticities are set to one, a value with no verified Canadian source, which is why

Section 8 sweeps it from 0.05 to 2.

5.3. Baseline equilibrium and verification

The baseline stationary equilibrium clears at $p^* = 1.000$ and $\tilde{p}_r^* = 1$ by normalisation, with an annual rent–price ratio of 0.039; relative excess demands are 0.000004 in the owner-occupied market and 0.000007 in the rental market, the lottery converges in 5 rounds and the whole equilibrium takes 8 seconds. Bellman residuals, measured as the relative difference between the interpolated value function and a re-optimisation on a 400-point asset grid at 400 random off-grid states, average 0.00067 for owners (99th percentile 0.6%, maximum 1.9%) and 0.00016 for renters; no household hits the forced-sale rule and 0.003% of household-periods use a clipped interpolated policy. Table B2 in Appendix B collects these diagnostics with the grid, tolerance, sample-size and seed variants, and Figure B1 plots them. The budget identities of every branch, the law of motion of nominal and real debt, the origination constraints, the terminal condition, the dynastic link, the lottery, the common random numbers, the monotonicity of the value functions and the single crossing of the tenure decision are unit-tested, and the zero-sum revaluation identity holds to machine precision in every transition.

Table 1 compares the model’s moments with the verified comparison points. The ownership rate is 0.63 overall, 0.38 under 35, 0.73 at 35–64 and 0.61 at 65 and over; 0.41 of households hold a mortgage against 0.39 in the Survey of Financial Security; the mean net worth of owners is 11.2 times that of renters under 35, against a median ratio of about 10 in the survey. Two moments are flagged as not supported by the comparison points. The ownership profile peaks at 0.86 at 55–64 and falls to 0.36 at 80–84, because households with a warm glow over a liquid estate sell late in life to consume and to bequeath cash, whereas Hou (2010) report a plateau near three quarters after 65; this is a known consequence of the preference specification and is discussed in Section 9. And 72% of first-time buyers receive a positive gift, well above the 28% quoted by Mirdamadi and Khalid (2023), because every parent in the model has a warm-glow motive and the gift is the only transfer available at entry. These discrepancies do not affect the mechanisms the paper studies but they bound what the magnitudes can claim.

Figure 2 shows the policy functions at the entry ages. The buy-minus-rent value crosses zero exactly once along cash at every age from 1 to 3 and every productivity state, which is the single-crossing property that Proposition 1 proves only on the hand-to-mouth region. At the crossing the buyer sets next-period liquid assets to zero for every state, so Assumption 4 holds for the marginal buyer; the marginal renter, however, saves, because in the quantitative economy renters accumulate towards a later purchase, which the benchmark rules out. This asymmetry is the origin of the scope restriction in Remark 1.

Table 1. Baseline stationary equilibrium: model moments and verified comparison points

Moment	Model	Comparison point (source)
Ownership rate, all households	0.631	about 0.66–0.69 (not a verified target); plateau ≈ 0.75 near 65 (Hou 2010)
Ownership rate, under 35 / 35–64 / 65+	0.38 / 0.73 / 0.61	steep before 40, plateau near 65 (Hou 2010)
Share of households with a mortgage	0.411	0.39 (SFS 2023)
Share of owners with a mortgage	0.651	–
Mean net worth owners / renters, under 35	11.2	median 457,100 / 44,000 ≈ 10.4 (SFS 2023); model median of renters = 0
Mean gift received at entry (5-year income units)	0.388	about \$80,000 (CIBC, 2021, via Mirdamadi–Khalid 2023)
Share of entrants receiving a positive gift	0.695	28% of first-time buyers received gifts (same source)
Mean estate received at age 55–59	0.316	–
Median house value / annual income (owners)	5.0	–
Mean LTV of new loans	0.605	–
Annual rent–price ratio	0.039	illustrative (landlord wedge ϕ)
Gini of net worth (all ages)	0.542	–
P(child owns at 35–39 parent renter / owner)	0.35 / 0.54	children of owners more likely to own (Mirdamadi–Khalid 2023)
Rank–rank slope of net worth (child at 55–59 vs parent at 55–59)	0.34	–
Aggregate mortgage debt / deposits (per age mass)	2.47 / 4.12	–

Model moments from results/data/baseline/summary.json (pooled cross-section of 6 periods, 1,152,000 household-periods). Comparison points are quoted from the verified sources listed in research/literature_matrix.csv (research/literature_review.md §1, §7) and are not fitted: the calibration is illustrative (Table B1). The young renters' median net worth is zero in the model (no liquid saving before purchase), so the median ratio is undefined; the mean ratio is reported.

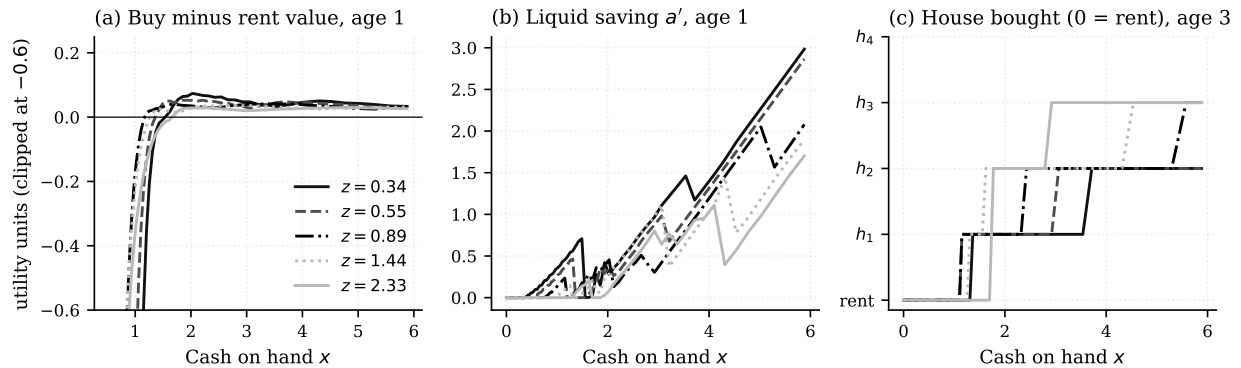


Figure 2. Policy functions on the cash-on-hand grid in the baseline stationary equilibrium. Panel (a) plots the value of buying minus the value of renting at age 1 (25–29) by productivity state, clipped at -0.6 ; the single sign change along cash is the ownership-entry threshold. Panel (b) plots liquid saving at age 1 and panel (c) the house size bought at age 3 (0 = rent). Source: results/data/baseline/policy_age{1,3}.csv.

6. Baseline simulations: inheritance, leverage and inequality

This section reports the baseline stationary equilibrium and the experiments that isolate the inheritance mechanism: life-cycle profiles, the entry threshold, equal-income wealth paths with different initial transfers, the dependence of inequality and mobility on transfers, the down-payment sweep in general and partial equilibrium, and the accounting of negative price shocks under leverage. All magnitudes are statements about this parameterisation.

6.1. Life-cycle profiles and the aggregate nominal position

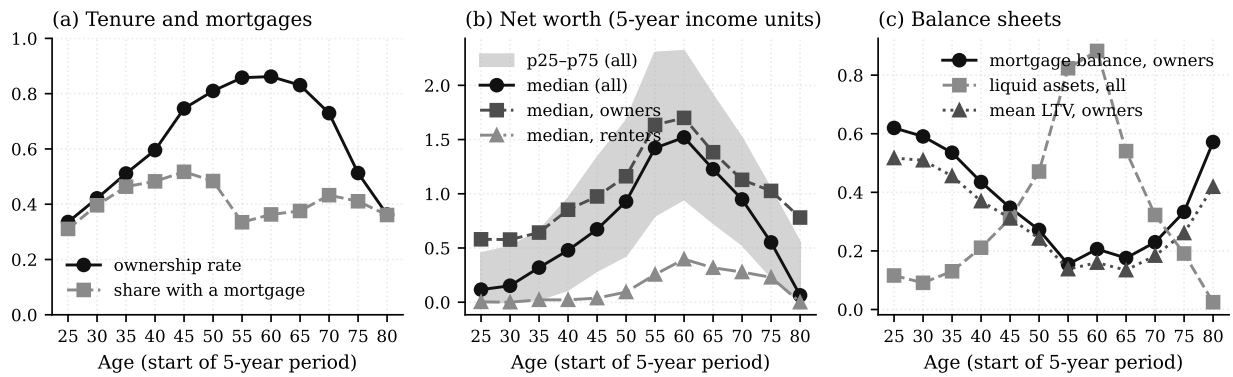


Figure 3. Baseline life-cycle profiles in the stationary general equilibrium: tenure and mortgage status (a), mean net worth of owners and renters (b) and balance sheets (c), by five-year age group. Units: average five-year labour income. Source: results/data/baseline/profiles_by_age.csv.

Figure 3 shows the life-cycle profiles. Ownership rises from 0.34 at 25–29 to 0.86 at 55–64 and falls to 0.36 at 80–84; a share 0.65 of owners carry a mortgage, with a mean loan-to-value ratio at origination of 0.60 and a median house value of 5.0 years of income. The net-worth gap between owners and renters, who have the same income process by construction, is 0.62 years of income at 30–34, 0.95 at 40–44, 1.57 at 55–59 and 1.03 at 70–74. The gap is housing equity carried forward: forced amortisation and the leverage multiplier of Proposition 4 accumulate it for owners while renters, whose median net worth under 35 is zero, save little before purchase. The balance-sheet panel contains the fact that governs the inflation results of Section 7: aggregate real mortgage debt is 2.47 per unit of age mass against real deposits of 4.12, so households are net nominal creditors by a ratio of 1.7 to one. Mortgage debt is concentrated among owners aged 30–50; deposits are held by the young saving for the down payment and, mostly, by owners aged 55–75 who have repaid their loans and are accumulating a liquid estate. The old-age sale of houses is the moment where the model departs from the Canadian profile, as noted in Section 5, and it also means that the estate is liquid: 55% of children receive a positive estate at 55–59, of mean size 0.32, and 70% receive a positive gift at entry, of mean size 0.39 (1.9 years of average income, median 0.22).

6.2. The ownership-entry threshold

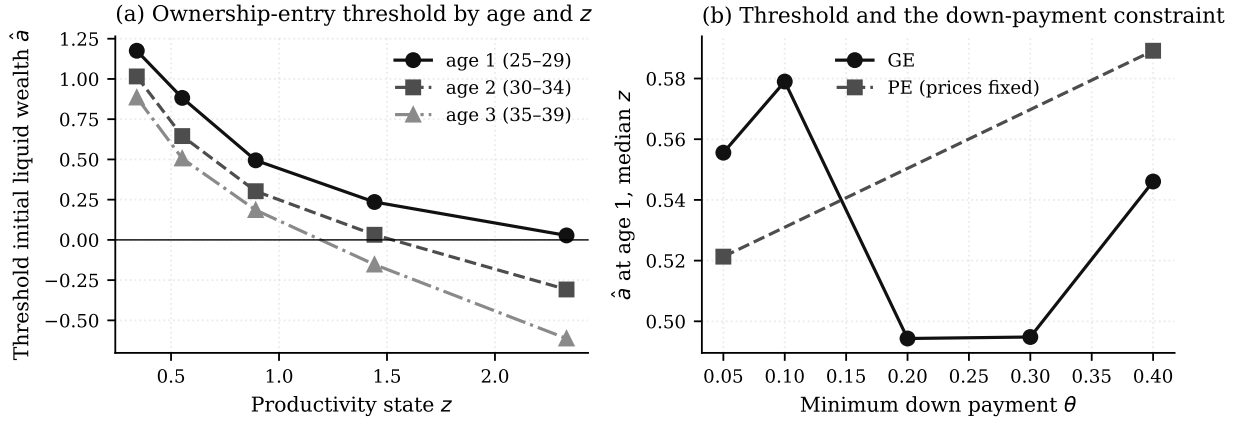


Figure 4. Ownership-entry thresholds (units: average five-year income). Panel (a): the smallest initial liquid wealth at which a renter buys, by age and productivity state, in the baseline. Panel (b): the age-1 threshold at the median productivity state as a function of the minimum down payment, in general equilibrium (prices re-solved) and at fixed prices. The grid sensitivity of this statistic is about ± 0.07 . Source: results/data/baseline/thresholds.csv, results/data/theta_sweep/*.json.

The quantitative counterpart of $\hat{b}$ is the smallest liquid wealth at which the age- j renter buys, $\hat{a}_j(z)$. Figure 4(a) plots it. At age 1 it is 1.18, 0.88, 0.49, 0.24 and 0.03 years of income for the five productivity states from the lowest to the highest; it falls with age, and at ages 2 and 3 it is negative for the two highest states (-0.31 and -0.61 at the top), meaning that those households buy out of income alone. The dependence on productivity is the feasibility component of Proposition 1 operating through the payment-to-income cap: at the lowest state the cap, not the loan-to-value constraint, binds, and the threshold is the cash needed to buy the smallest house with the largest loan the cap allows. The threshold is the least robust statistic of the paper, moving by $+0.07$ between the coarse and baseline grids and -0.02 between the baseline and fine grids (Table B2), which is why statements about it are confined to differences larger than that.

Panel (b) is the check of Proposition 2 and the first of the three discrepancies. At fixed prices the age-1 threshold at the median state is 0.52 with a 5% minimum down payment, 0.49 with 20% and 0.59 with 40%: it rises with θ between 20% and 40%, as the proposition predicts, but it is not lower at 5% than at 20%. The difference is within the grid sensitivity, and its economic source is the one named in Remark 1: with a small down payment the option of renting now and buying later is more valuable, which raises the renter's continuation value and the cash at which buying now is preferred. For the highest productivity state the threshold falls monotonically with θ at fixed prices (-0.13 , 0.03 , 0.24); for the lowest it is unchanged (1.18, 1.18, 1.18) because the loan-to-value constraint does not bind there. Proposition 2 is therefore supported where its binding-constraint and once-and-for-all premises hold and contradicted, within noise, where they do not. In general

equilibrium the price and rent responses add a second layer, and the threshold is 0.56, 0.58, 0.49, 0.49 and 0.55 for the five values of θ .

6.3. Equal-income wealth paths

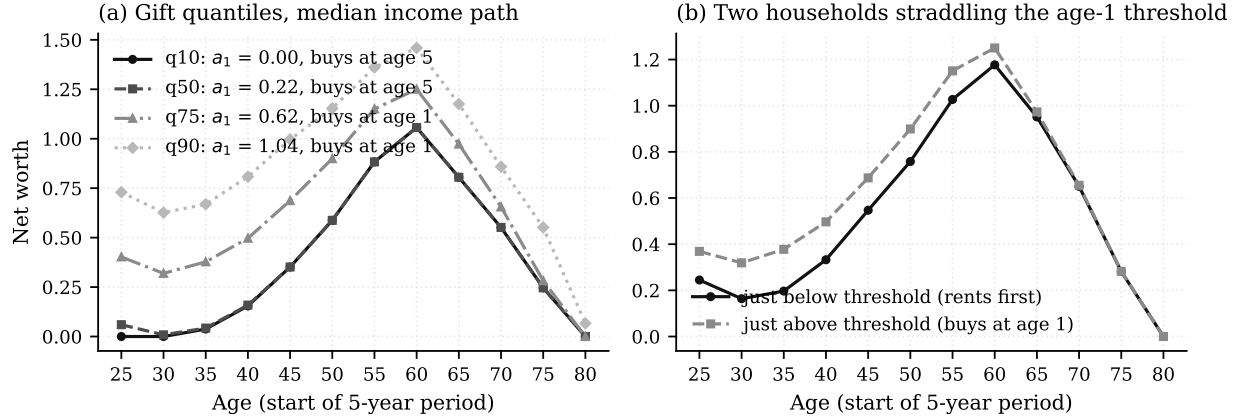


Figure 5. Net worth along a deterministic median-income path under baseline policies, for entrants with initial transfers at the 10th to 90th percentiles of the gift distribution (a) and for two entrants whose transfers straddle the age-1 threshold, $a_1 = 0.484$ and 0.504 (b). Units: average five-year income; the estate at 55–59 is set at the median lottery point. Source: results/data/baseline/equal_income_paths.csv.

Figure 5 isolates the mechanism of Propositions 4 and 5 by holding the income path fixed at the median and varying only the transfer at entry. Entrants with gifts at or below the median ($a_1 \leq 0.22$) rent until 45–49 and then buy the smallest house with the maximum loan; the entrant just above the age-1 threshold ($a_1 = 0.504$) buys at 25–29 and the one just below ($a_1 = 0.484$) at 40–44. Net worth of the two straddling households is 0.50 against 0.33 at 40–44 and 1.15 against 1.03 at 55–59: a 4% difference in endowment becomes a 49% difference in wealth at 40–44 and a 12% difference at 55–59, before the paths converge after retirement when both households sell. The threshold converts small endowment differences into two-decade wealth gaps through earlier leverage and forced amortisation; the sign is that of Proposition 4’s multiplier and the magnitude is illustrative.

6.4. Transfers, inequality and mobility

Figure 6 shows the same mechanism in the cross-section. Ownership at 25–29 is 0.02, 0.03, 0.18, 0.57 and 0.89 from the bottom to the top quintile of the transfer received; the bottom two quintiles receive nothing. The gap between the second and the fifth quintile narrows from 86 percentage points at 25–29 to 16 at 55–59 as the untransferred households buy later, but median net worth at 55–59 is still 1.06 in the second quintile against 2.31 in the fifth. Measured at the age at which the gift is made, 55–59, the probability that the child owns is 0.87 if the parent owns and 0.81 if the

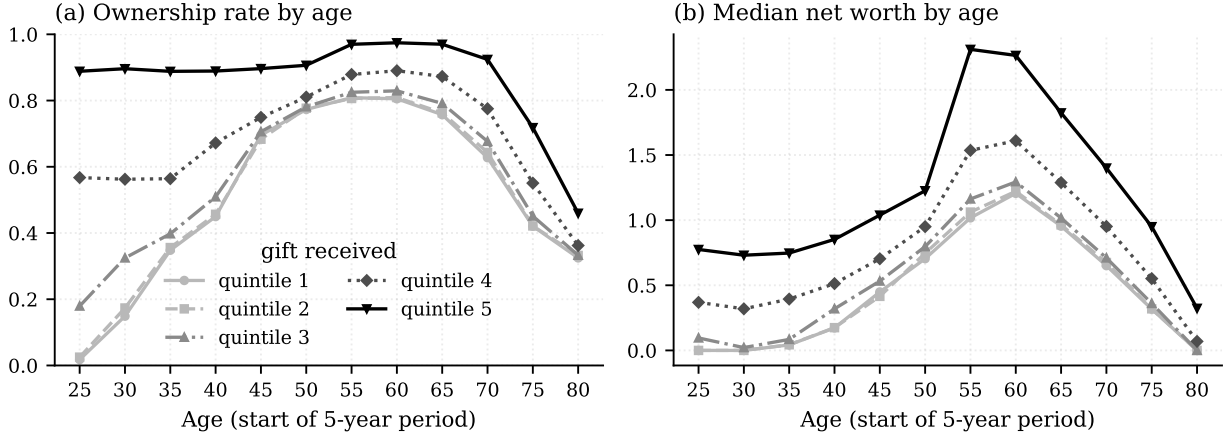


Figure 6. Ownership rate (a) and median net worth (b) by age and by quintile of the inter-vivos transfer received at entry, baseline cross-section. The bottom two quintiles receive no transfer. Source: results/data/baseline/by_gift_quintile.csv.

parent rents, so the persistence statistic of Proposition 5 is $\lambda_7 = 0.060$, a half-life of 0.25 generations; it is weak at that age because 86% of parents own by then. At 35–39 the same probabilities are 0.54 and 0.35, $\lambda_3 = 0.19$. The rank–rank slope of net worth between parents and children at 55–59 is 0.34 and the Gini of net worth is 0.54 over all ages and 0.38 at 55–59.

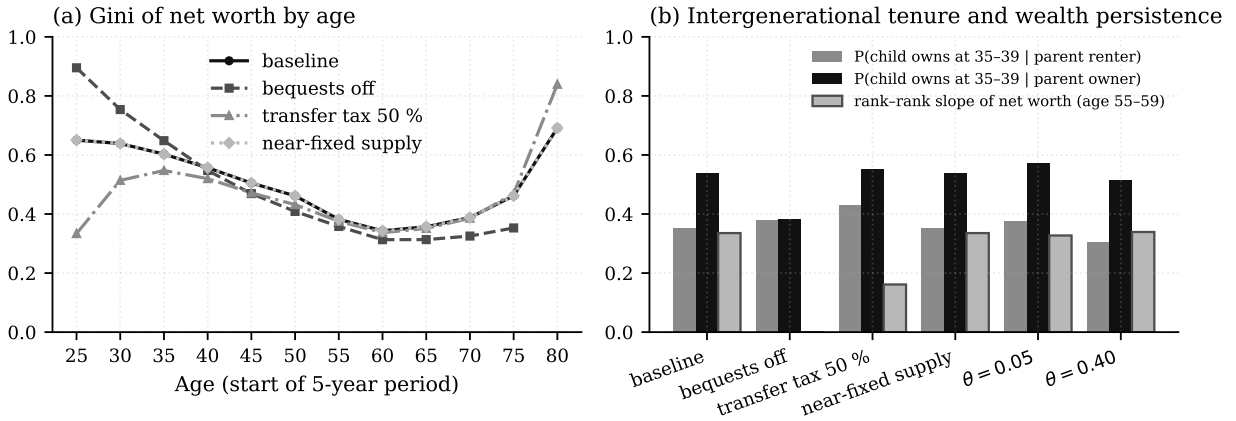


Figure 7. Inequality and mobility across stationary equilibria (general equilibrium unless stated): Gini of net worth by age (a) and intergenerational tenure and wealth persistence (b) for the baseline, without bequests, with a 50% rebated transfer tax, under near-fixed supply, and with minimum down payments of 5% and 40%. Source: results/data/{baseline,bequests,sensitivity}/*.json.

Table 2 and Figure 7 vary the transfers. Without bequests ($v = 0$) the price falls to 0.931, the rent rises by 7.7%, ownership falls to 0.581 (0.188 under 35, against 0.38), and the transition probabilities of the child become equal, $q_{RO} = 0.379$ and $q_{OO} = 0.381$, with a rank–rank slope of -0.01 : intergenerational persistence disappears entirely, as Proposition 5 requires, and here it is the

Table 2. Intergenerational persistence and inequality across stationary equilibria

Scenario	Eq.	p^*	Own.	q_{RO}	q_{OO}	λ_3	λ_7	rank–rank	Gini	gap ₄	gift	estate
Baseline	GE	1.000	0.631	0.35	0.54	0.19	0.060	0.34	0.54	0.95	0.39	0.32
Bequests off	GE	0.931	0.581	0.38	0.38	0.00	−0.003	−0.01	0.58	0.73	0.00	0.00
Bequests halved	GE	0.981	0.615	0.37	0.49	0.12	0.035	0.25	0.55	0.86	0.26	0.21
Transfer tax 20%	GE	1.005	0.642	0.39	0.57	0.18	0.054	0.28	0.53	0.96	0.28	0.22
Transfer tax 50%	GE	0.999	0.639	0.43	0.55	0.12	0.023	0.16	0.52	0.90	0.12	0.09
Intergenerational income correlation 0.4	GE	1.004	0.624	0.22	0.54	0.32	0.121	0.41	0.55	1.02	0.40	0.32
Earnings risk $\sigma = 0.10$	GE	0.963	0.672	0.31	0.49	0.19	0.043	0.32	0.51	0.59	0.30	0.22
Earnings risk $\sigma = 0.20$	GE	1.033	0.602	0.39	0.57	0.18	0.097	0.34	0.56	1.35	0.49	0.42
$\theta = 0.05$	GE	1.005	0.640	0.38	0.57	0.20	0.069	0.33	0.55	0.84	0.39	0.31
$\theta = 0.40$	GE	0.987	0.619	0.30	0.51	0.21	0.084	0.34	0.54	0.98	0.39	0.33
$\varepsilon = 0.05$	GE	1.000	0.631	0.35	0.54	0.19	0.060	0.34	0.54	0.95	0.39	0.32
$\varepsilon = 2$	GE	1.000	0.631	0.35	0.54	0.19	0.060	0.34	0.54	0.95	0.39	0.32
Deposit return +2 pp (renter investment)	GE	1.018	0.635	0.38	0.66	0.28	0.169	0.57	0.50	1.19	0.61	0.77
Real rate 1%	GE	1.042	0.601	0.31	0.48	0.17	0.063	0.26	0.55	0.90	0.36	0.24
Real rate 3%	GE	0.972	0.642	0.35	0.60	0.24	0.132	0.44	0.53	1.09	0.46	0.48
Trend inflation 0%	GE	1.011	0.640	0.35	0.54	0.19	0.058	0.33	0.55	0.91	0.39	0.30
Trend inflation 4%	GE	0.986	0.625	0.34	0.53	0.19	0.060	0.34	0.54	0.97	0.39	0.33
Landlord wedge $\phi = 0.02$	GE	0.991	0.607	0.33	0.52	0.19	0.070	0.33	0.55	0.96	0.39	0.31
Landlord wedge $\phi = 0.08$	GE	1.007	0.650	0.36	0.55	0.19	0.055	0.34	0.54	0.94	0.39	0.32

q_{RO} , q_{OO} : probability that the child owns at 35–39 given the parent’s tenure when the gift is made; $\lambda_3 = q_{OO} - q_{RO}$; λ_7 : the same difference measured at 55–59 (the Part A persistence statistic); rank–rank: slope of the child’s net-worth rank on the parent’s at the same age (55–59); gap₄: mean net-worth difference between owners and renters at 40–44 (equal income process by design); gift, estate: mean transfers received. [†] = not converged to tolerance. Source: results/data/group/name.json.

whole of it because incomes are independent across generations by design. The Gini rises to 0.58, since transfers lift the bottom of the young distribution more than the top, and the equal-income gap at 40–44 falls from 0.95 to 0.73. At fixed prices the same change lowers ownership by 12 percentage points; the general-equilibrium price fall offsets all but 5 of them. Halving the bequest motive gives intermediate values (price 0.981, $q_{OO} = 0.492$, rank–rank 0.25). The comparison with an intergenerational income correlation of 0.4 is the caveat of [Fagereng et al. \(2021\)](#) in model form: q_{RO} falls to 0.223 and λ_3 rises to 0.32 with the rank–rank slope at 0.41, so most of the tenure persistence beyond the baseline comes from income transmission, not from housing, and the model’s transfer channel should be read as one mechanism among several.

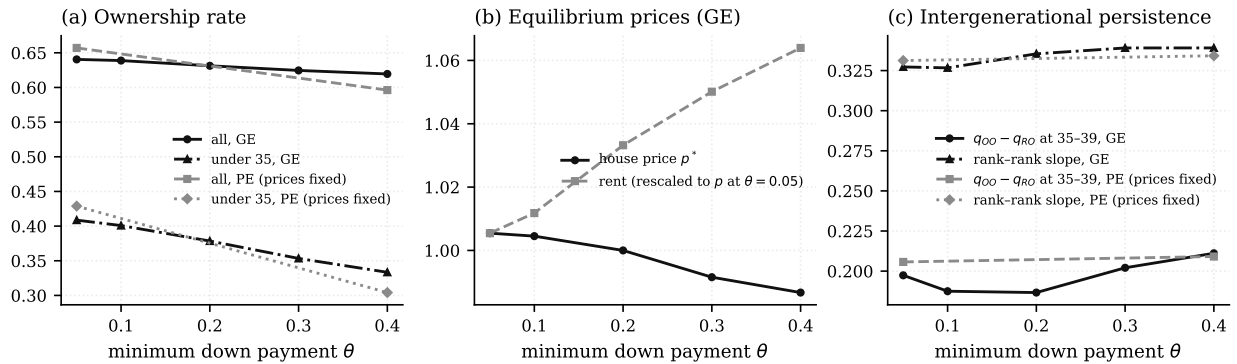
6.5. The down-payment constraint in general and partial equilibrium

Table 3 and Figure 8 report the down-payment sweep. In general equilibrium a minimum down payment of 5, 10, 20, 30 and 40% gives prices of 1.005, 1.005, 1.000, 0.992 and 0.987 and ownership rates of 0.640, 0.639, 0.631, 0.625 and 0.619, with under-35 ownership from 0.409 to 0.333. At fixed prices the 5% constraint raises ownership to 0.657 (0.429 under 35) and the 40% constraint lowers it to 0.596 (0.304): the price and rent responses absorb 64% and 66% of the partial-equilibrium ownership effect, the direction of Corollary 1 with a unit supply elasticity rather than a fixed stock. Mobility moves with the constraint, q_{RO} at 35–39 falling from 0.375 to 0.304 and λ_7 rising from 0.069 to 0.084, while the rank–rank slope is flat at 0.33–0.34: a tighter constraint lowers upward

Table 3. Stationary equilibria across credit, supply, segmentation and bequest scenarios

Scenario	Eq.	p^*	ρ/p	Own.	Own. <35	$\hat{a}_1$	q_{RO}	q_{OO}	rank–rank	Gini	gift
Baseline ($\theta = 0.20, \varepsilon = 1$)	GE	1.000	0.039	0.631	0.378	0.49	0.35	0.54	0.34	0.54	0.39
$\theta = 0.05$	GE	1.005	0.039	0.640	0.409	0.56	0.38	0.57	0.33	0.55	0.39
$\theta = 0.10$	GE	1.005	0.039	0.639	0.401	0.58	0.37	0.56	0.33	0.54	0.39
$\theta = 0.30$	GE	0.992	0.039	0.625	0.353	0.49	0.32	0.52	0.34	0.54	0.39
$\theta = 0.40$	GE	0.987	0.039	0.619	0.333	0.55	0.30	0.51	0.34	0.54	0.39
$\theta = 0.05$, prices fixed	PE	1.000	0.039	0.657	0.429	0.52	0.38	0.58	0.33	0.54	0.39
$\theta = 0.40$, prices fixed	PE	1.000	0.039	0.596	0.304	0.59	0.28	0.49	0.33	0.54	0.39
DSTI cap $\psi = 0.30$	GE	0.991	0.039	0.627	0.362	0.59	0.36	0.53	0.33	0.54	0.39
DSTI cap $\psi = 0.50$	GE	1.006	0.039	0.634	0.382	0.46	0.34	0.53	0.33	0.54	0.39
$\varepsilon = 0.05$ (near-fixed supply)	GE	1.000	0.039	0.631	0.378	0.49	0.35	0.54	0.34	0.54	0.39
$\varepsilon = 0.25$	GE	1.000	0.039	0.631	0.378	0.49	0.35	0.54	0.34	0.54	0.39
$\varepsilon = 0.5$	GE	1.000	0.039	0.631	0.378	0.49	0.35	0.54	0.34	0.54	0.39
$\varepsilon = 2$	GE	1.000	0.039	0.631	0.378	0.49	0.35	0.54	0.34	0.54	0.39
No segmentation ($\sigma = 0$)	GE	1.000	0.039	0.631	0.378	0.49	0.35	0.54	0.34	0.54	0.39
Half segmentation ($\sigma = 0.5$)	GE	1.000	0.039	0.631	0.378	0.49	0.35	0.54	0.34	0.54	0.39
Bequests off ($v = 0$)	GE	0.931	0.039	0.581	0.188	0.32	0.38	0.38	−0.01	0.58	0.00
Bequests halved ($v = 1$)	GE	0.981	0.039	0.615	0.325	0.44	0.37	0.49	0.25	0.55	0.26
Transfer tax 20% (rebated)	GE	1.005	0.039	0.642	0.461	0.51	0.39	0.57	0.28	0.53	0.28
Transfer tax 50% (rebated)	GE	0.999	0.039	0.639	0.482	0.48	0.43	0.55	0.16	0.52	0.12
Bequests off, prices fixed	PE	1.000	0.039	0.512	0.106	0.48	0.36	0.36	0.00	0.59	0.00

GE = general equilibrium (prices, estate lottery and rebate re-solved; supply scales fixed at the baseline normalisation); PE = partial equilibrium with prices fixed at the baseline (household block and distribution re-solved). † = not converged to tolerance (residuals in results/data/diagnostics.csv). ρ/p = annual rent–price ratio; $\hat{a}_1$ = age-1 ownership-entry threshold at the median productivity state (5-year income units); q_{RO}, q_{OO} = probability that the child owns at 35–39 given the parent’s tenure at the gift; rank–rank = slope of the child’s net-worth rank on the parent’s (both at 55–59); gift = mean transfer received at entry. Source files: results/data/group/name.json.

**Figure 8.** Down-payment sweep: ownership rate (a), equilibrium prices (b) and intergenerational persistence (c) as functions of the minimum down payment θ , in general equilibrium (prices, lottery and rebate re-solved) and with prices fixed at the baseline. Source: results/data/theta_sweep/*.json.

tenure mobility without changing the rank persistence of wealth, because the latter is dominated by the size of transfers rather than by who crosses the threshold. The payment-to-income cap has a smaller effect in the same direction (ownership 0.627 and 0.634 for $\psi = 0.30$ and 0.50).

6.6. Leverage and negative price shocks: accounting

Proposition 4 is an accounting statement and is checked as one, on the baseline cross-section with prices fixed after the drop. Existing owners carry a mean loan-to-value ratio of 0.27 and no household is underwater after a 10, 20 or 30% fall in the house price, because the first annuity payment, a fifth of the loan, is made within the purchase period and the balance carried across five-year periods never exceeds the underwater threshold x^* ; the mean loss relative to net worth is 12, 24 and 37% for the three drops. New borrowers evaluated at origination, with a mean loan-to-value of 0.60, are 0, 40 and 48% underwater, consistent with $x^* = 1 - 0.8/0.95 \approx 0.16$ at the 80% cap with a 5% selling cost, and lose 17, 35 and 52% of net worth. The five-year period therefore understates peak exposure, a limitation noted in Section 9; the general-equilibrium counterpart of a price fall, a persistent rise in the real rate, is in Section 7.6.

7. Surprise inflation, mortgage contracts and a real-rate shock

7.1. Design

The economy starts in the baseline stationary equilibrium and at $t = 0$ the price level turns out 10% above expectation over the shock period (a level surprise, about two percentage points per year for five years), or 10% above in each of two consecutive periods with the second known at $t = 0$ (a persistent surprise); inflation then returns to target and expectations are correct thereafter. The transition is solved with perfect foresight of the post-shock price path over 70 years, durable stocks, and the three contracts: a fixed-rate mortgage whose nominal payment is fixed for the remaining term, the five-year term whose rate resets at the next period boundary, and a stylised adjustable-rate mortgage repriced within the period so that its balance is not eroded. Each transition is reported next to its partial-equilibrium counterpart, the same shock with prices fixed at the stationary values, and is labelled with its maximum market-clearing residual: the fixed-rate level surprise stalled at 2.2×10^{-3} after a Jacobian refresh and is labelled as not converged, while the term and adjustable level surprises and the three persistent surprises converged to 6.1×10^{-4} – 1.2×10^{-3} . For a level surprise the fixed-rate and five-year-term economies are identical at the five-year frequency, since both erode the balance once and both face the same post-shock rate, and every impact statistic of the two runs agrees to two decimals ($p_0 = -0.45$ against -0.47%); the pair therefore bounds the solver’s price noise at about 0.02 percentage points. The price paths are back within 0.1% of the

stationary state after 20 years and the residual at the terminal date is 3×10^{-4} . Zero-sum revaluation holds to machine precision.

Table 4. Unanticipated 10% price-level surprise: impact effects by mortgage contract and persistence

Scenario	Eq.	p_0	ρ_0	Δ_{own}	$\Delta_{\text{own PE}}$	erosion	dep. loss	CEV o/m	CEV r	CEV all	CEV PE
FRM, level surprise	GE [†]	-0.45	-1.14	-0.02	-0.20	0.206	0.481	0.63	-1.00	-0.64	-0.68
5-year term, level	GE	-0.47	-1.03	-0.02	-0.20	0.206	0.481	0.63	-1.00	-0.64	-0.68
ARM, level	GE	-0.79	-1.15	-0.11	-0.47	0.000	0.481	-1.03	-0.96	-1.14	-1.19
FRM, persistent (2 periods)	GE	-1.88	-0.38	-0.18	-2.17	0.206	0.481	1.14	-1.06	-0.52	-0.53
5-year term, persistent	GE	-1.49	0.04	-0.34	-2.03	0.206	0.481	0.46	-1.09	-0.74	-0.73
ARM, persistent	GE	-1.86	-0.18	-0.39	-2.34	0.000	0.481	-1.21	-1.04	-1.24	-1.24
FRM, level, rigid rents ($\eta_p = 0$)	GE	-0.46	7.48	-0.03	-2.88	0.206	0.481	0.55	-0.92	-0.62	-0.35
FRM, level, unindexed wages ($\eta_w = 0$)	GE [†]	-3.86	-3.21	-0.63	-3.18	0.206	0.481	-2.40	-3.58	-3.33	-3.45
FRM, persistent, real-rate response	GE	-7.25	-2.29	-0.12	-2.64	0.206	0.481	1.43	-0.31	0.14	-1.32
5-year term, persistent, real-rate response	GE	-6.93	-1.91	-0.26	-2.94	0.206	0.481	0.20	-0.32	-0.25	-1.67
FRM, level, $\varepsilon = 0.05$	GE [†]	-0.50	-1.08	-0.01	-0.19	0.206	0.481	0.63	-0.99	-0.64	-0.68
FRM, level, no segmentation	GE	-0.39	-1.47	-0.16	-0.20	0.206	0.481	0.66	-0.95	-0.60	-0.68
FRM, level, static expectations (A6)	A6	-0.36	-1.11	0.02	-0.20	0.206	0.481	0.65	-1.01	-0.64	-0.68

Perfect-foresight transitions (GE: prices clear every period; [†] = not converged to the 1.5×10^{-3} per-period tolerance (stalled at the Monte-Carlo noise floor; maximum residual in results/data/inflation/name.json); A6 = static expectations, stationary continuation values). Columns: real house price on impact (%; the impact is also the peak deviation in every run), real rent on impact (%), change of the ownership rate on impact (pp) in GE and with prices fixed (PE), aggregate real debt erosion and real deposit loss per unit mass of one age (the lender's revaluation is their difference with the opposite sign), consumption-equivalent welfare of households alive at the shock (owners with a mortgage, renters, all; %), and the all-household CEV with prices fixed. Level surprise: CPI 10% above expectation in the shock period, inflation back to target afterwards; persistent: two periods. Source: results/data/inflation/*.json and *_welfare.csv.

7.2. Impact: the real house price falls

Table 4 and Figure 9 give the central negative result of the paper. A level surprise lowers the real house price on impact by 0.45% under the fixed-rate and term contracts and by 0.79% under the adjustable contract, and lowers the real rent by 1.14 and 1.15%; a persistent surprise lowers the price by 1.88% (fixed rate), 1.49% (term) and 1.86% (adjustable). Proposition 9 predicts a rise. The reason is the one stated in Corollary 2 and visible in the aggregate balance sheet of Section 6: households are net nominal creditors. The surprise erodes real mortgage debt by 0.21 per unit of age mass under the fixed-rate contract (and not at all under the adjustable one) but erodes real deposits by 0.48, so the lender gains 0.27 (0.48 under the adjustable contract) and aggregate household wealth falls; the demand for housing, on both the extensive and the intensive margin, falls with it. The direction is the corollary's; the magnitude of the rent response is not, because the corollary's unit demand makes the two markets mirror images of a fixed number of units, whereas here the wealth loss lowers the demand for rented services as well, so both prices fall.

The ownership rate barely moves in general equilibrium, by -0.02 percentage points on impact for the fixed-rate level surprise against -0.20 with prices fixed, -0.11 against -0.47 for the adjustable contract, and -0.18 against -2.17 for the persistent fixed-rate surprise: the price adjustment absorbs 76–92% of the partial-equilibrium ownership response, the composition result of Corollary 1 in a durable-stock economy whose impact supply elasticity is $\varepsilon \delta_H \approx 0.05$. The price

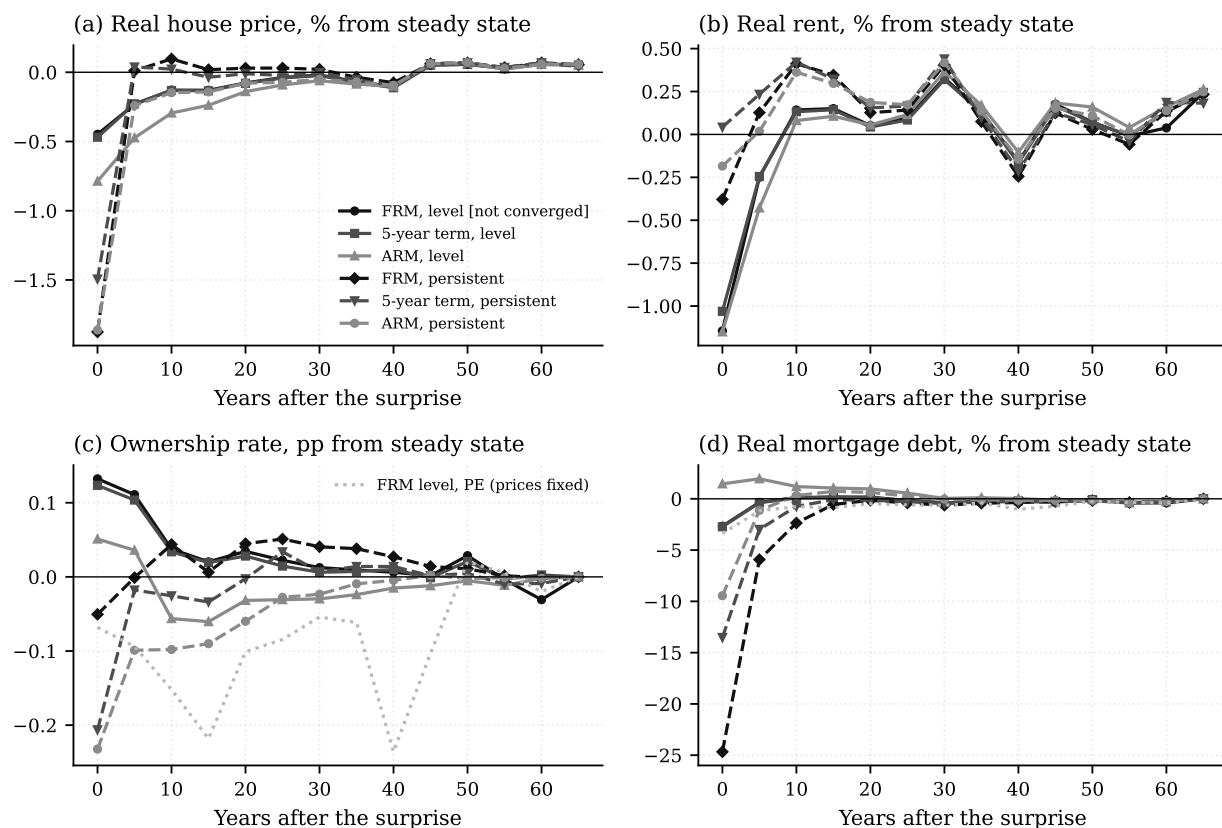


Figure 9. Perfect-foresight transitions after a 10% unanticipated price-level surprise (general equilibrium unless labelled): real house price (a), real rent (b), ownership rate (c) and real mortgage debt (d), in deviation from the stationary state, for the three contracts and the two persistence profiles; the dotted line in (c) is the fixed-rate level surprise with prices fixed. The fixed-rate level run is reported with its residual. Source: results/data/inflation/*_path.csv.

paths return with 1 sign change for the level surprise and 3 for the persistent one, all after the impact and all smaller than 0.1%; the cohort-crowding oscillation found in a myopic simulation of the benchmark does not appear under perfect foresight with durable stocks.

The persistent surprise differs from the level surprise through the nominal rate expected for the second period, which jumps from 0.219 to 0.341 per period. Real debt falls on impact by 24.7% under the fixed-rate contract, whose nominal payments are fixed through both inflation periods, by 13.5% under the term contract and by 9.4% under the adjustable contract, where the fall is lower origination rather than erosion: the qualifying payment at the higher nominal rate tightens the payment-to-income cap, ownership falls by 2.17 to 2.34 percentage points with prices fixed and by 0.18 to 0.39 in general equilibrium. This is Proposition 3's tilt operating along the transition through the nominal constraint, and it is why the persistent surprise lowers the price four times more than the level surprise.

7.3. Redistribution and the contract ordering

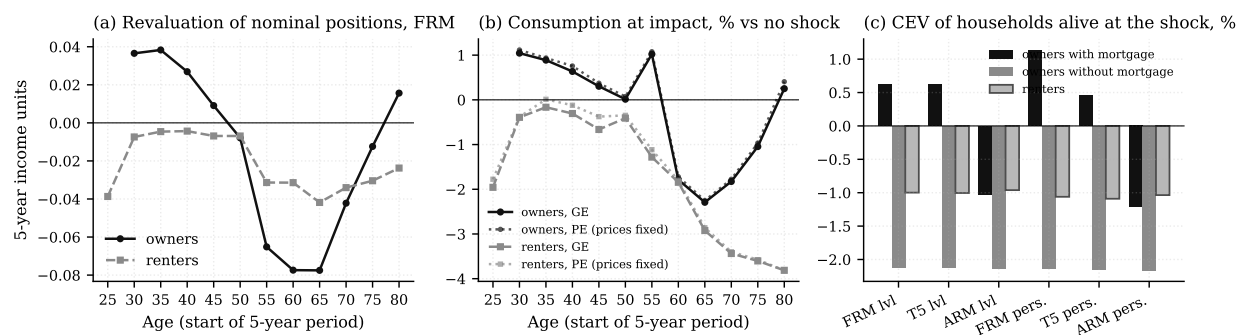


Figure 10. Redistribution in the shock period of the fixed-rate level surprise: revaluation of nominal positions by age and tenure (a), consumption at impact relative to the no-shock path (b), and consumption-equivalent welfare of households alive at the shock by age and tenure (c), general equilibrium. Source: results/data/inflation/frm_level_impact.csv, frm_level_welfare.csv.

Figure 10 shows who gains and who loses within the fixed-rate level surprise. Owners aged 30–44 gain on their nominal position, up to 0.04 of a five-year income, and raise impact consumption by 0.6 to 1.0%; owners aged 60–74, deposit-rich and debt-free, lose 0.04 to 0.08 and cut consumption by 1.8 to 2.3%; renters lose 0.02 on average and cut consumption by 1.3%. In lifetime terms, owners with a mortgage gain 0.63% of consumption (71% of them are better off), owners without a mortgage lose 2.12%, renters lose 1.00% and all households alive at the shock lose 0.64% on average, against 0.68% with prices fixed: the price response barely changes the incidence because prices move little. This is the age and tenure pattern of [Doepke and Schneider \(2006\)](#) and [Meh and Terajima \(2011\)](#), with the model-specific addition that the losers include the young renters saving for a down payment.

The contract ordering of Proposition 8 is confirmed. For a level surprise the fixed-rate and term contracts give identical gains to mortgaged owners (0.63%) while the adjustable contract turns the gain into a loss (−1.03%, 7% better off), since no balance is eroded and the deposit loss remains. For a persistent surprise, mortgaged owners gain 1.14% under the fixed-rate contract (73% better off), 0.46% under the five-year term (66%) and lose 1.21% under the adjustable contract, the ordering fixed rate above term above adjustable, with debt-free owners losing about 2.13% and renters about 1.06% in every case. The Canadian renewable term is thus intermediate, as the institutional reading of Section 2 suggested, and the gap between it and the long fixed-rate contract opens only when the surprise outlasts the term.

7.4. Wage indexation, rent rigidity and a real-rate response

Three variants test the conditions of Proposition 8. With unindexed nominal wages ($\eta_w = 0$; the run stalled at 2.0×10^{-3} and is labelled) the one-period real wage loss of 0.08 per household dwarfs the nominal revaluation, the price falls by 3.86% and the rent by 3.21%, consumption falls by 4.4% on impact, and every group loses, mortgaged owners by 2.40% with 0% better off: condition (18) fails for every indebted owner once the real-income loss exceeds the debt erosion. With rigid rents for sitting tenants ($\eta_p = 0$), incumbents receive a one-period transfer of 0.01 from landlords, the market rent for new tenants rises by 7.48% on impact because incumbents stay put, ownership falls by 2.88 percentage points with prices fixed but only 0.03 in general equilibrium, and the renters' welfare loss improves only from −1.00 to −0.92%: the hedge of Pallotti et al. (2024) is present but, at the five-year frequency, small. With a real-rate response to the persistent surprise ($\phi_r = 0.5$, a rise of five percentage points per period in the two surprise periods), the rate response dominates: the price falls by 7.25% (fixed rate) and 6.93% (term), mortgaged owners gain 1.43% under the fixed-rate contract but only 0.20% under the five-year term (70 against 56% better off), because the renewal at the higher real rate claws back most of the term borrowers' gain, exactly as Proposition 8 states for $\phi_r > 0$; debt-free owners and renters are cushioned by the higher deposit return, and the average household gains 0.14% under the fixed-rate contract and loses 0.25% under the term. The distinction between the Canadian term and the long fixed-rate contract therefore matters precisely when the surprise is persistent and monetary policy reacts.

7.5. Segmentation, supply elasticity and expectations

Three further variants check the structural assumptions of Proposition 9. Without segmentation ($\sigma = 0$, one stock) the price falls by 0.39% against 0.45% with segmented stocks and the rent by 1.47%, with ownership moving by −0.16 percentage points in general equilibrium against −0.20 with prices fixed and identical welfare: the neutrality of Proposition 7 does not carry over, for the

reason given in Remark 2, and segmentation itself adds only about a seventh to the price response. With near-fixed long-run supply ($\varepsilon = \varepsilon_r = 0.05$; the run stalled at 3.1×10^{-3} and is labelled) the price falls by 0.50% against 0.45%, and ownership moves by -0.01 percentage points against -0.19 at fixed prices: the long-run elasticity barely matters on impact because the stock is durable and the shock-period supply is essentially fixed in both cases, so the price absorbs the demand shift and the composition result of Corollary 1 is what the numbers show. Under static expectations (Assumption 6, with stationary continuation values and prices clearing period by period) the price falls by 0.36% against 0.45% under perfect foresight, ownership moves by 0.02 against -0.02 percentage points, welfare agrees to two decimals and the gift-age transition probability q_{RO} falls from 0.806 to 0.797 in both: the expectation scheme is immaterial for the size of the effects in this calibration, and the sign of the price response is negative under both. Mobility on impact is as Proposition 9 predicts in direction and negligible in size, q_{RO} at the gift age falling to 0.797 and λ_7 rising from 0.060 to 0.069; the age-1 threshold at the median state rises from 0.49 to 0.53, by 8%, through the erosion of the savings of would-be buyers rather than through a higher price, the third channel named in Remark 3.

7.6. A real-rate shock and leverage

The general-equilibrium counterpart of a house-price fall is a persistent rise in the real rate: two percentage points per year for twenty years, unanticipated at $t = 0$, from the stationary equilibrium of each of three down-payment regimes, with fixed-rate contracts keeping their origination rate. The runs for $\theta = 0.20$ and 0.40 converged (9.5×10^{-4} and 1.1×10^{-3}); the run for $\theta = 0.05$ diverged from a cold start and, after a warm start and a damped tâtonnement pre-phase, stalled at 8.8×10^{-3} , so its numbers are indicative only. The real house price falls on impact by 15.54% at $\theta = 0.20$ and 14.61% at $\theta = 0.40$ (16.16% in the unconverged run), overshoots by 6.9% when the rate returns to 2% twenty years later, and the rent falls by 5.32%. Ownership on impact moves by 0.01 and 0.16 percentage points in general equilibrium against -4.11 and -1.77 with prices fixed: the ranking of the partial-equilibrium responses across θ (-5.56 , -4.11 , -1.77 for 5, 20 and 40%) is the leverage-exposure ordering of Proposition 4, and general equilibrium removes it. Owners lose 0.11 of a five-year income of net worth on impact and cut consumption by 6.0%; renters' net worth rises slightly and their consumption falls by 3.9%. Welfare is nevertheless positive on average, 2.33% for households alive at the shock, because net nominal creditors gain from the higher real return, which dominates the housing-wealth loss: mortgaged owners gain 0.97% (65% better off, shielded by the fixed rate while their deposits earn more), debt-free owners 2.96% and renters 2.98%. The price fall lowers the entry threshold (from 0.55 to 0.49 at the median state for $\theta = 0.40$) and raises upward mobility (q_{RO} at 35–39 from 0.351 to 0.380 at $\theta = 0.20$). Leverage thus amplifies the housing-equity loss, as the accounting says, but with fixed-rate contracts, net creditor positions and

no default the channel operates through consumption and entry rather than through foreclosure.

8. Supply elasticity, policy and welfare

8.1. Supply elasticity and the down-payment constraint

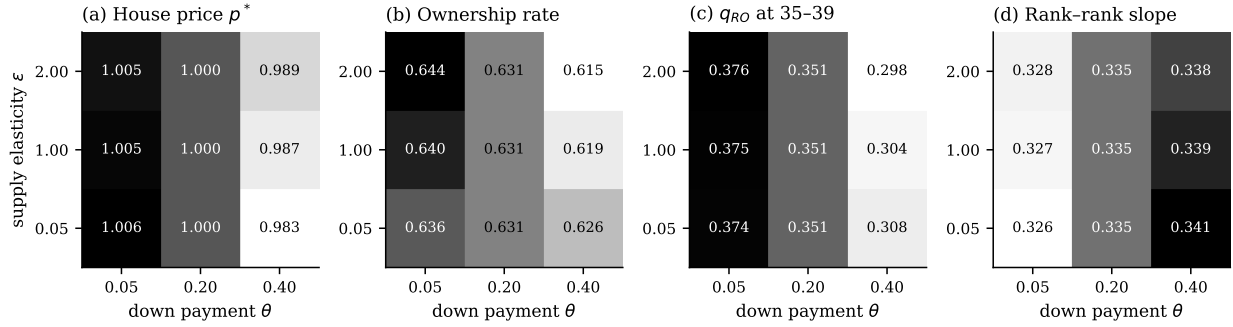


Figure 11. Stationary general equilibria across the long-run supply elasticity $\varepsilon = \varepsilon_r$ and the minimum down payment θ : house price (a), ownership rate (b), probability that a renter’s child owns at 35–39 (c) and rank–rank slope of net worth (d). Source: `results/data/supply/*.json`, `theta_sweep/*.json`.

Changing the supply elasticity alone leaves the baseline unchanged, because every supply curve passes through the normalisation point, so the elasticity matters only together with a demand shift, and Figure 11 crosses it with the down-payment constraint. With a 5% minimum down payment the price is 1.006 under near-fixed supply and 1.005 under an elasticity of two, with ownership 0.636 against 0.644; with a 40% minimum the price is 0.983 against 0.989 and ownership 0.626 against 0.615. The direction is that of Corollary 1, inelastic supply converting more of the demand shift into price and less into quantity, but the magnitudes are small because the demand shift from θ is itself small: the payment-to-income cap binds for low-income entrants whatever the down payment. Mobility is insensitive to the elasticity at these shifts, q_{RO} at 35–39 being 0.374 against 0.376 at $\theta = 0.05$ and 0.308 against 0.298 at $\theta = 0.40$. The conjecture with which the fourth contribution started, that supply elasticity has a first-order effect on intergenerational mobility through the entry threshold, is not supported in this calibration; what the elasticity does govern, in the transitions of Section 7, is bounded by the depreciation rate.

8.2. Search for multiple stationary equilibria

Proposition 6 leaves multiplicity open when condition (C) fails, and the benchmark’s ladder channel, bequeathed equity rising with the price, is the candidate. Figure 12 scans the owner-market excess demand over prices from 0.80 to 1.25. In the baseline it falls monotonically from 0.69 to -0.43 with a single sign change at $p = 1.000$, and the ownership rate along the clearing curve falls from 0.73

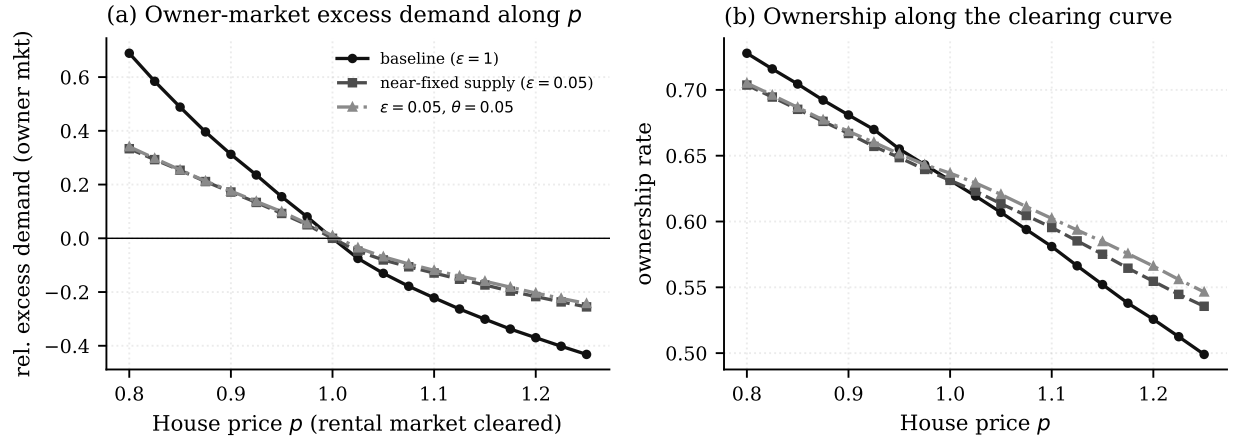


Figure 12. Search for multiple stationary equilibria: owner-market excess demand along the house price with the rental market cleared at each point and the estate lottery held at its baseline value (a), and the ownership rate along the clearing curve (b), for the baseline, near-fixed supply and near-fixed supply with a 5% minimum down payment. Source: results/data/multiplicity/price_scan_*.csv.

to 0.50: $d\mu/dp < 0$ everywhere, the analogue of (C), holds with a wide margin. Under near-fixed supply the excess demand runs from 0.33 to -0.26 with one root, and under near-fixed supply with a 5% down payment from 0.34 to -0.24 with one root at 1.000. There is no evidence of the [Stein \(1995\)](#) multiplicity in the scanned range. The scan holds the lottery fixed, and the lottery feedback is the ladder channel itself, so the scan understates it; but the slope of excess demand around the root, -3.1 per unit of log price, is an order of magnitude larger than the lottery feedback measured across rounds, and at the equilibrium the lottery-consistent solution coincides with the root. Uniqueness is a numerical statement about this calibration, not a theorem.

8.3. A rebated transfer tax

The policy experiment taxes gifts and estates at 20 or 50% and rebates the proceeds lump-sum to entering households, with the rebate an equilibrium object. Ownership rises to 0.642 and 0.639, and under-35 ownership to 0.461 and 0.482 against 0.38 in the baseline, because the rebate gives every entrant liquid wealth; upward tenure mobility rises (q_{RO} at 35–39 0.392 and 0.429), the rank–rank slope falls to 0.28 and 0.16, the Gini to 0.53 and 0.52, and the mean gift falls to 0.28 and 0.12 (Tables 3 and 2). The price is nearly unchanged (1.005 and 0.999). A rebated transfer tax therefore raises youth ownership and mobility in this economy by moving liquidity to the moment at which the down-payment constraint binds; it is a comparative-statics statement about stationary equilibria, not an optimal-policy result, and the welfare numbers below concern entrants only.

Table 5. Welfare of entering households across stationary equilibria (consumption-equivalent variation, %)

Scenario	Q1	Q2	Q3	Q4	Q5	marginal	low z_1	high z_1	all
$\theta = 0.05$	0.55	0.54	0.43	0.26	-0.08	0.30	0.54	0.20	0.34
$\theta = 0.10$	0.41	0.40	0.33	0.19	-0.05	0.24	0.41	0.14	0.26
$\theta = 0.30$	-0.20	-0.21	-0.22	-0.07	0.21	-0.13	-0.23	0.08	-0.10
$\theta = 0.40$	-0.39	-0.40	-0.44	-0.25	0.32	-0.33	-0.44	0.07	-0.23
DSTI cap $\psi = 0.30$	-0.01	0.00	0.01	-0.02	0.09	-0.08	-0.08	0.21	0.02
DSTI cap $\psi = 0.50$	-0.01	-0.02	-0.05	-0.06	-0.12	-0.01	0.03	-0.16	-0.05
Bequests off ($v = 0$) [‡]	8.87	9.09	10.11	12.08	15.53	12.82	4.41	20.16	11.14
Transfer tax 20% (rebated)	10.85	9.64	7.20	5.64	4.02	5.74	16.92	2.58	7.47
Transfer tax 50% (rebated)	15.39	13.79	10.68	8.52	5.89	8.58	24.44	2.87	10.85
$\varepsilon = 0.05$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\varepsilon = 2$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
No segmentation	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Real rate 1%	0.68	0.51	-0.03	-0.68	-2.25	-0.86	-0.31	-1.18	-0.35
Real rate 3%	0.42	0.62	1.25	2.23	4.46	2.32	1.46	3.06	1.80
Trend inflation 4%	-0.11	-0.11	-0.08	-0.01	0.23	-0.08	-0.16	0.21	-0.01
Deposit return +2 pp	5.23	5.69	7.34	9.97	15.40	10.27	7.76	11.92	8.73
Ownership premium $\chi = 0.05$ [‡]	0.81	0.82	0.86	0.95	1.08	0.94	0.77	1.09	0.91
Transaction costs low	-0.22	-0.22	-0.21	-0.14	-0.17	-0.14	-0.29	-0.08	-0.19
Transaction costs high	-0.07	-0.08	-0.11	-0.16	-0.10	-0.18	-0.03	-0.12	-0.10

λ^{CE} such that scaling non-housing consumption by $1 + \lambda^{CE}$ in every period of the baseline makes an age-1 household with state (z_1, a_1) as well off as in the alternative stationary equilibrium: $V_1^{alt} = (1 + \lambda^{CE})^{(1-\alpha)(1-\gamma)} U^{base} + \Phi^{base}$, where U^{base} (discounted flow utility) and Φ^{base} (warm glow) come from cohort simulations under baseline policies. Columns: quintiles of the transfer received at entry (averaged over z_1 with stationary weights), households within $\pm 10\%$ of the age-1 ownership threshold (“marginal”), lowest and highest z_1 , all. Comparisons are between stationary equilibria, not along transitions. [‡] = the scenario changes a preference parameter, so the number compares value functions under different utilities and is not a welfare statement. Source: results/data/welfare/welfare_all.csv.

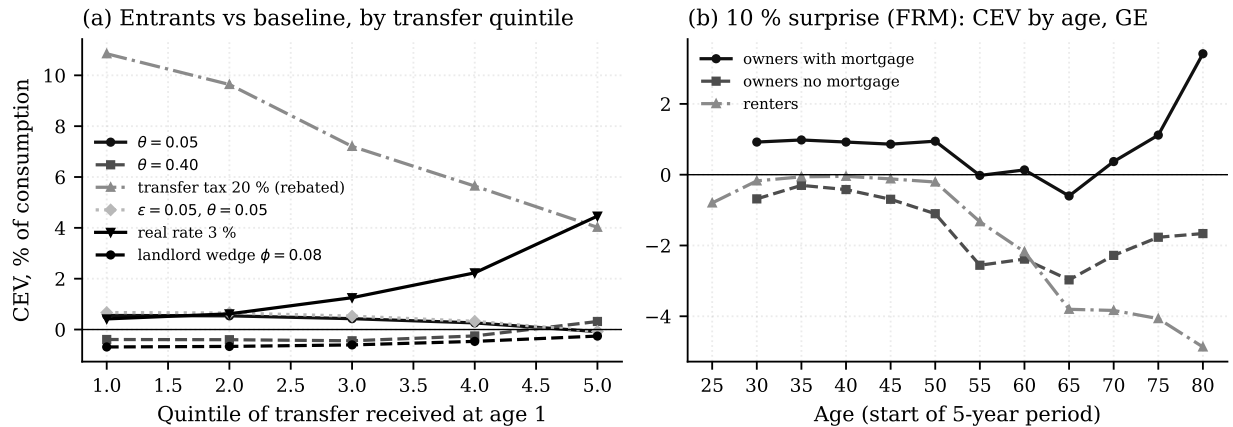


Figure 13. Consumption-equivalent welfare. Panel (a): entering households in alternative stationary equilibria relative to the baseline, by quintile of the transfer received. Panel (b): households alive at a 10% fixed-rate level surprise, by age and tenure, general equilibrium. Source: results/data/welfare/welfare_all.csv, results/data/inflation/frm_level_welfare.csv.

8.4. Welfare

Table 5 and Figure 13 report the consumption-equivalent variation (21) of entering households across stationary equilibria, by quintile of the transfer received, for households within 10% of the age-1 threshold, and overall. Only scenarios that keep the preference parameters are welfare comparisons; the rows for the bequest motive and for the preference variants are listed as a reminder that they compare value functions under different utilities (“bequests off” shows a large positive number only because the warm-glow disutility disappears from the objective). A 5% minimum down payment raises the welfare of entrants by 0.34% overall, 0.55% in the bottom two gift quintiles, 0.30% for marginal households and -0.08% in the top quintile, which pays the higher price; a 40% minimum gives -0.23% overall, -0.40% at the bottom, -0.33% for marginal households and 0.32% at the top. The welfare of marginal households, unsigned in the theory, is thus positive for a looser constraint and negative for a tighter one in this calibration. The rebated transfer tax raises entrants’ welfare by 7.47% and 10.85%, decreasing in the gift quintile (10.85 and 15.39% at the bottom against 4.02 and 5.89% at the top); this is the welfare of entrants, and the parents who pay the tax are not in the comparison. Near-fixed supply with a 5% down payment gives 0.42% against 0.29% with an elasticity of two. A real rate of 1 or 3% gives -0.35 and 1.80%; a landlord wedge of 2 or 8% gives 0.65 and -0.54% ; low and high earnings risk give 10.25 and -10.83% ; a 2-percentage-point deposit premium, the renter’s outside investment return, gives 8.73%, rising with the gift quintile. The numerical variants (tolerance, sample size, seed) give variations of at most 0.03%, which bounds the noise of the welfare measure. For transitions, the households alive at a level surprise lose 0.64% on average under the fixed-rate contract and 1.14% under the adjustable one, and gain 2.33% in the real-rate shock (Section 7).

9. Sensitivity, limitations and conclusion

9.1. Sensitivity

Table 2 and Table B3 in Appendix B report the sensitivity of the stationary equilibrium to rates, trend inflation, the rent wedge, earnings risk, the outside asset, preferences and transaction costs; every variant is a full re-resolution of the general equilibrium. A real rate of 1 or 3% gives prices of 1.042 and 0.972, ownership of 0.601 and 0.642, mean estates of 0.24 and 0.48, λ_7 of 0.063 and 0.132 and rank–rank slopes of 0.26 and 0.44: a higher real rate raises saving, estates and persistence. Trend inflation of 0 or 4% gives prices of 1.011 and 0.986 and ownership of 0.640 and 0.625; at fixed prices, 4% inflation lowers under-35 ownership by 4.9 percentage points through the nominal tilt of the constant-payment mortgage and the nominal payment-to-income cap, which is Proposition 3 in partial equilibrium, and the general-equilibrium price fall offsets 65% of it. A landlord wedge

of 2 or 8% gives ownership of 0.607 and 0.650. Earnings risk with annual innovation standard deviations of 0.10 and 0.20 gives ownership of 0.672 and 0.602, Ginis of 0.51 and 0.56, mean gifts of 0.30 and 0.49 and λ_7 of 0.043 and 0.097: more risk raises precautionary transfers and persistence while lowering ownership. A deposit premium of two percentage points, the renter's investment return, raises the age-1 threshold at the median state to 0.75, estates to 0.77 and the rank–rank slope to 0.57: a better outside asset delays purchase but raises estates and persistence, which is the mechanism of [Nekoei and Seim \(2023\)](#) in reverse. Annual discount factors of 0.94 and 0.98 give prices of 0.933 and 1.075 and ownership of 0.581 and 0.681; $\gamma = 3$ gives 0.646; a housing share of 0.25 gives a price of 1.101; low and high transaction costs give prices of 1.044 and 0.962; a 5% ownership premium gives 0.644. None of these variants changes the qualitative results: persistence requires transfers, prices absorb credit shifts, the surprise lowers the real price.

The numerical variants bound the precision of the statistics. Coarse grids move ownership by -0.5 percentage points, the price by -0.8% , the Gini by 0.000, the age-1 threshold by $+0.07$ and λ_7 by $+0.017$; fine grids move ownership by 0.4 points, the price by -0.1% and the threshold by -0.02 . A tolerance of 10^{-5} returns the same best iterate; a panel of 240,000 dynasties and an alternative seed move ownership by -0.1 and -0.1 points, the price by 0.0 and 0.0% , λ_7 by $+0.004$ and $+0.001$ and the rank–rank slope by -0.002 and $+0.000$. The ownership rate, prices, the Gini and the rank–rank slope are robust; the age-1 threshold and λ_7 are the least robust statistics and are quoted accordingly. Mean Bellman residuals of owners lie between 4.0×10^{-4} (fine grids) and 1.9×10^{-3} ($\gamma = 3$ and coarse grids) across the 49 robustness runs.

9.2. Limitations

The calibration is illustrative, no moment is fitted and the supply elasticity has no verified Canadian source, so magnitudes are statements about this parameterisation. The ownership profile falls after 65 because the warm glow is defined over a liquid estate, whereas Canadian owners keep their houses; a bequest motive over housing, or the slow dissaving of [Nakajima and Telyukova \(2020\)](#), would keep the old in their houses, raise the housing share of estates and, since the house would then be inherited at a market price, strengthen the ladder channel that the multiplicity scan found weak. Seventy percent of entrants receive a gift, against 28% in the Canadian data, because every parent has the same motive; heterogeneous motives would concentrate transfers and raise the dispersion the model understates. There is no aggregate risk, no default, no survival risk and no endogenous real rate; the estate is anticipated as an unconditional lottery and held at its stationary value along transitions; the adjustable-rate contract is a stylised within-period repricing; and the five-year period understates peak leverage, so that no household is ever underwater and the leverage channel operates through consumption and entry rather than foreclosure. Monte-Carlo noise bounds market clearing to about 5×10^{-4} in stationary equilibria and 10^{-3} along transitions, and four of

the 16 transitions are reported with residuals above tolerance. On the theory side, single crossing of the tenure decision is proved only on the hand-to-mouth region, the comparative statics in the down payment hold for a once-and-for-all tenure decision, price neutrality without segmentation holds under unit demand, and the impact proposition covers one period under static expectations and pure debtor positions. The general-equilibrium sign of the inflation response is governed by a density-weighted net nominal position that the quantitative model computes but that no verified Canadian source pins down; [Meh and Terajima \(2011\)](#) report net nominal positions by age and tenure, and a calibration that targeted them would be the natural next step.

9.3. Conclusion

The paper asked under what conditions inherited wealth, mortgage leverage, nominal debt, housing supply and down-payment constraints generate persistent owner–renter wealth inequality among households with the same income process, and what an unanticipated inflation changes. The benchmark answers the first part analytically. Ownership requires a transfer above a threshold that exceeds the down-payment bound by the consumption of the marginal buyer; leverage multiplies the exposure of terminal wealth to the resale price without changing the break-even; the parent’s housing equity is pivotal for the child’s tenure over an interval of income draws whose probability mass is the intergenerational persistence, positive only if the house is net wealth at the end of life, and a half-life rather than a trap; the stationary equilibrium exists and is unique when the entry-threshold channel dominates the ladder channel; and with a fixed stock, credit and transfers move prices and composition, not the ownership rate. The quantitative economy supports these statements in their stated scope. Transfers generate a mid-life wealth gap of about one year of income among equal-income households and are the whole of the model’s intergenerational persistence, which disappears when the bequest motive is removed; prices absorb two thirds to nine tenths of what credit constraints would do to ownership at fixed prices; and no multiplicity appears, even under near-fixed supply.

For the second part the benchmark was wrong in sign and the paper says so. A surprise erodes mortgages and deposits alike, and households in this economy hold more of the latter: the real house price falls, aggregate welfare falls, and the gains of mortgaged owners under fixed-rate and renewable-term contracts come from debt-free owners, renters and, through the lender, from rentiers. The benchmark’s prediction of a price rise is restored only under the assumption that the mortgage is the households’ sole nominal position, and the corollary that replaces it ties the sign of the price response to a density-weighted net nominal position of the parents whose children are at the tenure margin. Two further benchmark results fail outside their premises, price neutrality without segmentation under unit demand and the monotonicity of the entry threshold in the down payment for a once-and-for-all tenure decision, and both have been restated under those premises. What

survives of the inflation analysis in general equilibrium is the accounting and the contract ordering: fixed-rate at least as good as the renewable term and both better than the adjustable contract for the indebted, with the term losing its advantage exactly when the surprise is persistent and the real rate responds, which is the Canadian case. The natural continuation is a calibration that targets the net nominal positions of Canadian households by age and tenure and a bequest motive over housing, since the sign of the general-equilibrium response and the strength of the ladder channel both turn on them.

References

- Adam, Klaus and Junyi Zhu**, “Price-Level Changes and the Redistribution of Nominal Wealth across the Euro Area,” *Journal of the European Economic Association*, 2016, 14 (4), 871–906.
- Aiyagari, S. Rao**, “Uninsured Idiosyncratic Risk and Aggregate Saving,” *Quarterly Journal of Economics*, 1994, 109 (3), 659–684.
- Allen, Jason, Robert Clark, and Jean-François Houde**, “Search Frictions and Market Power in Negotiated-Price Markets,” *Journal of Political Economy*, 2019, 127 (4), 1550–1598.
- Auclert, Adrien**, “Monetary Policy and the Redistribution Channel,” *American Economic Review*, 2019, 109 (6), 2333–2367.
- Benetton, Matteo, Marianna Kudlyak, and John Mondragon**, “Dynastic Home Equity,” SSRN Working Paper, SSRN 2022.
- Benhabib, Jess, Alberto Bisin, and Shenghao Zhu**, “The Distribution of Wealth and Fiscal Policy in Economies with Finitely Lived Agents,” *Econometrica*, 2011, 79 (1), 123–157.
- Blickle, Kristian and Martin Brown**, “Borrowing Constraints, Home Ownership and Housing Choice: Evidence from Intra-Family Wealth Transfers,” *Journal of Money, Credit and Banking*, 2019, 51 (2–3), 539–580.
- Boar, Corina**, “Dynastic Precautionary Savings,” *Review of Economic Studies*, 2021, 88 (6), 2735–2765.
- Borri, Nicola and Pietro Reichlin**, “Optimal Taxation with Home Ownership and Wealth Inequality,” *Review of Economic Dynamics*, 2021, 40, 64–84.
- Bounajm, Fares and Austin McWhirter**, “Modelling Canadian Mortgage Debt and Payments in a Semi-Structural Model,” Bank of Canada Staff Analytical Note 2024-1, Bank of Canada 2024.
- Campbell, John Y. and João F. Cocco**, “Household Risk Management and Optimal Mortgage Choice,” *Quarterly Journal of Economics*, 2003, 118 (4), 1449–1494.
- , **Nuno Clara, and João F. Cocco**, “Structuring Mortgages for Macroeconomic Stability,” *Journal of Finance*, 2021, 76 (5), 2525–2576.
- Canada Mortgage and Housing Corporation**, “Residential Mortgage Industry Report, Spring 2026,” CMHC 2026.

- Chambers, Matthew, Carlos Garriga, and Don E. Schlagenhauf**, “Accounting for Changes in the Homeownership Rate,” *International Economic Review*, 2009, 50 (3), 677–726.
- Charles, Kerwin Kofi and Erik Hurst**, “The Transition to Home Ownership and the Black-White Wealth Gap,” *Review of Economics and Statistics*, 2002, 84 (2), 281–297.
- Cloyne, James, Clodomiro Ferreira, and Paolo Surico**, “Monetary Policy when Households have Debt: New Evidence on the Transmission Mechanism,” *Review of Economic Studies*, 2020, 87 (1), 102–129.
- Davis, Morris A. and Jonathan Heathcote**, “The Price and Quantity of Residential Land in the United States,” *Journal of Monetary Economics*, 2007, 54 (8), 2595–2620.
- Daysal, N. Meltem, Michael F. Lovenheim, and David N. Wasser**, “The Intergenerational Transmission of Housing Wealth,” NBER Working Paper 31669 31669, National Bureau of Economic Research 2023.
- Diamond, Peter A.**, “National Debt in a Neoclassical Growth Model,” *American Economic Review*, 1965, 55 (5), 1126–1150.
- Díaz, Antonia and María José Luengo-Prado**, “The Wealth Distribution with Durable Goods,” *International Economic Review*, 2010, 51 (1), 143–170.
- Doepke, Matthias and Martin Schneider**, “Inflation and the Redistribution of Nominal Wealth,” *Journal of Political Economy*, 2006, 114 (6), 1069–1097.
- Engelhardt, Gary V. and Christopher J. Mayer**, “Intergenerational Transfers, Borrowing Constraints, and Saving Behavior: Evidence from the Housing Market,” *Journal of Urban Economics*, 1998, 44 (1), 135–157.
- Fagereng, Andreas, Magne Mogstad, and Marte Rønning**, “Why Do Wealthy Parents Have Wealthy Children?,” *Journal of Political Economy*, 2021, 129 (3), 703–756.
- Favilukis, Jack, Pierre Mabille, and Stijn Van Nieuwerburgh**, “Affordable Housing and City Welfare,” *Review of Economic Studies*, 2023, 90 (1), 293–330.
- , **Sydney C. Ludvigson, and Stijn Van Nieuwerburgh**, “The Macroeconomic Effects of Housing Wealth, Housing Finance, and Limited Risk Sharing in General Equilibrium,” *Journal of Political Economy*, 2017, 125 (1), 140–223.
- Fisher, Irving**, “The Debt-Deflation Theory of Great Depressions,” *Econometrica*, 1933, 1 (4), 337–357.

- Floetotto, Max, Michael Kirker, and Johannes Stroebe**, “Government Intervention in the Housing Market: Who Wins, Who Loses?,” *Journal of Monetary Economics*, 2016, 80, 106–123.
- Gabaix, Xavier, Jean-Michel Lasry, Pierre-Louis Lions, and Benjamin Moll**, “The Dynamics of Inequality,” *Econometrica*, 2016, 84 (6), 2071–2111.
- Garriga, Carlos, Finn E. Kydland, and Roman Šustek**, “Mortgages and Monetary Policy,” *Review of Financial Studies*, 2017, 30 (10), 3337–3375.
- Gervais, Martin**, “Housing Taxation and Capital Accumulation,” *Journal of Monetary Economics*, 2002, 49 (7), 1461–1489.
- Gete, Pedro and Michael Reher**, “Mortgage Supply and Housing Rents,” *Review of Financial Studies*, 2018, 31 (12), 4884–4911.
- Glaeser, Edward and Joseph Gyourko**, “The Economic Implications of Housing Supply,” *Journal of Economic Perspectives*, 2018, 32 (1), 3–30.
- Godbout, Claudia, Adam Su, and Yang Xu**, “How Will Mortgage Payments Change at Renewal? An Updated Analysis,” Bank of Canada Staff Analytical Note 2025-21, Bank of Canada 2025.
- Greenwald, Daniel L. and Adam Guren**, “Do Credit Conditions Move House Prices?,” *American Economic Review*, 2025, 115 (10), 3559–3596.
- Grossmann, Volker, Benjamin Larin, Hans Torben Löfflad, and Thomas Steger**, “Distributional Consequences of Surging Housing Rents,” *Journal of Economic Theory*, 2021, 196, 105275.
- Guiso, Luigi and Tullio Jappelli**, “Private Transfers, Borrowing Constraints, and Timing of Homeownership,” *Journal of Money, Credit, and Banking*, 2002, 34 (2), 315–339.
- Guren, Adam M., Arvind Krishnamurthy, and Timothy J. McQuade**, “Mortgage Design in an Equilibrium Model of the Housing Market,” *Journal of Finance*, 2021, 76 (1), 113–168.
- Gyourko, Joseph, Christopher Mayer, and Todd Sinai**, “Superstar Cities,” *American Economic Journal: Economic Policy*, 2013, 5 (4), 167–199.
- Halket, Jonathan and Matteo Pignatti Morano di Custoza**, “Homeownership and the Scarcity of Rentals,” *Journal of Monetary Economics*, 2015, 76, 107–123.
- **and Santhanagopalan Vasudev**, “Saving Up or Settling Down: Home Ownership over the Life Cycle,” *Review of Economic Dynamics*, 2014, 17 (2), 345–366.

- Henderson, J. Vernon and Yannis M. Ioannides**, “A Model of Housing Tenure Choice,” *American Economic Review*, 1983, 73 (1), 98–113.
- Hou, Feng**, “Homeownership over the Life Course of Canadians: Evidence from Canadian Censuses of Population,” Statistics Canada Analytical Studies Branch Research Paper 325 325, Statistics Canada 2010.
- Huggett, Mark**, “Wealth Distribution in Life-Cycle Economies,” *Journal of Monetary Economics*, 1996, 38 (3), 469–494.
- Iacoviello, Matteo**, “House Prices, Borrowing Constraints, and Monetary Policy in the Business Cycle,” *American Economic Review*, 2005, 95 (3), 739–764.
- **and Marina Pavan**, “Housing and Debt over the Life Cycle and over the Business Cycle,” *Journal of Monetary Economics*, 2013, 60 (2), 221–238.
- Kaas, Leo, Georgi Kocharkov, Edgar Preugschat, and Nawid Siassi**, “Low Homeownership in Germany—A Quantitative Exploration,” *Journal of the European Economic Association*, 2021, 19 (1), 128–164.
- Kaplan, Greg, Kurt Mitman, and Giovanni L. Violante**, “The Housing Boom and Bust: Model Meets Evidence,” *Journal of Political Economy*, 2020, 128 (9), 3285–3345.
- Kartashova, Katya and Xiaoqing Zhou**, “How Do Mortgage Rate Resets Affect Consumer Spending and Debt Repayment? Evidence from Canadian Consumers,” Bank of Canada Staff Working Paper 2020-18, Bank of Canada 2020.
- Kiyotaki, Nobuhiro, Alexander Michaelides, and Kalin Nikolov**, “Winners and Losers in Housing Markets,” *Journal of Money, Credit and Banking*, 2011, 43 (2–3), 255–296.
- **and John Moore**, “Credit Cycles,” *Journal of Political Economy*, 1997, 105 (2), 211–248.
- Knoll, Katharina, Moritz Schularick, and Thomas Steger**, “No Price Like Home: Global House Prices, 1870–2012,” *American Economic Review*, 2017, 107 (2), 331–353.
- Kuhn, Moritz, Moritz Schularick, and Ulrike I. Steins**, “Income and Wealth Inequality in America, 1949–2016,” *Journal of Political Economy*, 2020, 128 (9), 3469–3519.
- Linneman, Peter and Susan Wachter**, “The Impacts of Borrowing Constraints on Homeownership,” *Real Estate Economics*, 1989, 17 (4), 389–402.
- Meh, Césaire A. and Yaz Terajima**, “Inflation, Nominal Portfolios, and Wealth Redistribution in Canada,” *Canadian Journal of Economics*, 2011, 44 (4), 1369–1402.

- , **José-Víctor Ríos-Rull, and Yaz Terajima**, “Aggregate and Welfare Effects of Redistribution of Wealth under Inflation and Price-Level Targeting,” *Journal of Monetary Economics*, 2010, 57 (6), 637–652.
- Mian, Atif and Amir Sufi**, “House Prices, Home Equity-Based Borrowing, and the US Household Leverage Crisis,” *American Economic Review*, 2011, 101 (5), 2132–2156.
- Mirdamadi, Michael and Aisha Khalid**, “Parents and Children in the Canadian Housing Market: Does Parental Property Ownership Increase the Likelihood of Homeownership for their Adult Children?,” Statistics Canada, Housing Statistics in Canada, Catalogue 46-28-0001 2023.
- Nakajima, Makoto and Irina A. Telyukova**, “Home Equity in Retirement,” *International Economic Review*, 2020, 61 (2), 573–616.
- Nardi, Mariacristina De**, “Wealth Inequality and Intergenerational Links,” *Review of Economic Studies*, 2004, 71 (3), 743–768.
- Nekoei, Arash and David Seim**, “How Do Inheritances Shape Wealth Inequality? Theory and Evidence from Sweden,” *Review of Economic Studies*, 2023, 90 (1), 463–498.
- Ortalo-Magné, François and Sven Rady**, “Housing Market Dynamics: On the Contribution of Income Shocks and Credit Constraints,” *Review of Economic Studies*, 2006, 73 (2), 459–485.
- Pallotti, Filippo, Gonzalo Paz-Pardo, Jiri Slacalek, Oreste Tristani, and Giovanni L. Violante**, “Who Bears the Costs of Inflation? Euro Area Households and the 2021–2023 Shock,” *Journal of Monetary Economics*, 2024, 148, 103671.
- Paz-Pardo, Gonzalo**, “Homeownership and Portfolio Choice over the Generations,” *American Economic Journal: Macroeconomics*, 2024, 16 (1), 207–237.
- Piketty, Thomas and Gabriel Zucman**, “Capital is Back: Wealth-Income Ratios in Rich Countries 1700–2010,” *Quarterly Journal of Economics*, 2014, 129 (3), 1255–1310.
- Rognlie, Matthew**, “Deciphering the Fall and Rise in the Net Capital Share: Accumulation or Scarcity?,” *Brookings Papers on Economic Activity*, 2015, 2015 (1), 1–69.
- Saiz, Albert**, “The Geographic Determinants of Housing Supply,” *Quarterly Journal of Economics*, 2010, 125 (3), 1253–1296.
- Samuelson, Paul A.**, “An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money,” *Journal of Political Economy*, 1958, 66 (6), 467–482.

Sommer, Kamila and Paul Sullivan, “Implications of US Tax Policy for House Prices, Rents, and Homeownership,” *American Economic Review*, 2018, *108* (2), 241–274.

—, —, **and Randal Verbrugge**, “The Equilibrium Effect of Fundamentals on House Prices and Rents,” *Journal of Monetary Economics*, 2013, *60* (7), 854–870.

Statistics Canada, “Survey of Financial Security, 2023,” *The Daily*, 29 October 2024 2024.

Stein, Jeremy C., “Prices and Trading Volume in the Housing Market: A Model with Down-Payment Effects,” *Quarterly Journal of Economics*, 1995, *110* (2), 379–406.

Appendices

A. Proofs

Throughout, $R = 1 + r$, d , δ_2 , δ_3 and e_3 are as in (2), the identity (3) holds, and the hand-to-mouth consumption levels c_j^o and terminal resources W_3^o are those of Section 3.2. An owner who bought one unit at price p pays $D_2 = \delta_2 p$ at stage 2 and $D_3 = \delta_3 p$ at stage 3, and sells at stage 3 for $(1 - \kappa_s)p$ in a stationary state.

A.1. Preliminary lemmas

Lemma 1 (Stage-3 split). *A stage-3 household with resources W_3 chooses $c_3 = W_3/(1 + v)$ and $b' = vW_3/(1 + v)$.*

Proof. Maximise $\ln c_3 + v \ln b'$ subject to $c_3 + b' = W_3$: the first-order condition $1/c_3 = v/b'$ with the constraint gives the result; the objective is strictly concave. $\square$

Lemma 2 (Hand-to-mouth values). *Under Assumptions 1–4, for $b \in \mathcal{B}$ the optimal plan of a renter is $n_1 = n_2 = 0$ and that of an owner is $B = (1 - \theta)p$, $n_1 = n_2 = 0$. The value functions are*

$$V^O(b) = \ln c_1^O(b) + \chi + \beta \ln c_2^O + \beta^2(1 + v) \mathbb{E} \ln W_3^O(y_3) + K, \quad (22)$$

$$V^R(b) = \ln c_1^R(b) + \beta \ln c_2^R + \beta^2(1 + v) \mathbb{E} \ln W_3^R(y_3) + K, \quad (23)$$

with $K = \beta^2[v \ln v - (1 + v) \ln(1 + v)]$, and $V^O(b) - V^R(b) = \ln c_1^O(b) - \ln c_1^R(b) + \Delta$.

Proof. Using Lemma 1, the stage-3 continuation is $(1 + v) \ln W_3 + K$. For a renter the problem is to maximise $\ln(b + y_1 - \rho - n_1) + \beta \ln(y_2 - \rho + n_1 R - n_2) + \beta^2(1 + v) \mathbb{E} \ln(y_3 + n_2 R)$ over $n_1, n_2 \geq 0$ (n_1 is chosen at stage 1 and n_2 at stage 2, but since c_2 is deterministic given n_1 the two-stage problem equals this static problem). The objective is strictly concave in (n_1, n_2) and the constraint set is convex, so the Karush–Kuhn–Tucker conditions are necessary and sufficient. At $n_1 = n_2 = 0$ they read $1/c_1^R \geq \beta R/c_2^R$ and $1/c_2^R \geq \beta R(1 + v) \mathbb{E}[1/W_3^R]$, which hold strictly by (12). For an owner, the choice variables are (B, n_1, n_2) with $0 \leq B \leq (1 - \theta)p$; lowering B by ε lowers c_1 by ε and raises c_2 by $\varepsilon \delta_2/(1 - \theta)$ and W_3 by $\varepsilon \delta_3/(1 - \theta)$. The KKT condition for $B = (1 - \theta)p$ is

$$\frac{1}{c_1^O} \geq \beta \frac{\delta_2}{1 - \theta} \frac{1}{c_2^O} + \beta^2(1 + v) \frac{\delta_3}{1 - \theta} \mathbb{E} \left[\frac{1}{W_3^O} \right].$$

By (12), $\beta/c_2^O < 1/(Rc_1^O)$ and $\beta^2(1 + v) \mathbb{E}[1/W_3^O] < \beta/(Rc_2^O) < 1/(R^2 c_1^O)$, so the right-hand side is strictly below $\frac{1}{c_1^O} \cdot \frac{\delta_2/R + \delta_3/R^2}{1 - \theta} = \frac{1}{c_1^O}$ by (3). The conditions for $n_1 = n_2 = 0$ are as for the renter. Substituting the corner allocations gives (22)–(23), and the difference follows from (13). $\square$

A.2. Threshold, comparative statics and tilt

Proof of Proposition 1. Let $D(b) \equiv V^O(b) - V^R(b) = \ln c_1^O(b) - \ln c_1^R(b) + \Delta$ on $\mathcal{B}$ (Lemma 2). D is continuous, $D(b) \rightarrow -\infty$ as $b \downarrow dp - y_1$ (since $c_1^O \rightarrow 0$ while $c_1^R \rightarrow dp - \rho > 0$), and $D'(b) = 1/c_1^O(b) - 1/c_1^R(b) > 0$ because $c_1^O(b) = c_1^R(b) - (dp - \rho) < c_1^R(b)$ by Assumption 5. As $b \rightarrow \infty$, $\ln(c_1^O/c_1^R) \rightarrow 0$ so $D(b) \rightarrow \Delta > 0$. Hence D has exactly one zero $\hat{b}$, at which $c_1^O/c_1^R = \Lambda$, i.e. $b + y_1 - dp = \Lambda(b + y_1 - \rho)$, which gives (14); the two identities follow by substitution. If $dp \leq \rho$, $c_1^O \geq c_1^R$ so $D(b) \geq \Delta > 0$ for all feasible b . $\square$

Remark 4 (Proof gap: single crossing outside the hand-to-mouth region). Proposition 1 characterises the tenure decision on $\mathcal{B}$, where Assumption 4 holds. For $b > \bar{b}$ the household saves and the values are not the closed forms (22)–(23); the envelope derivative is still $1/c_1^O(b)$ for each tenure, so $V^O - V^R$ is increasing wherever $c_1^O(b) < c_1^R(b)$, but this has not been shown at every crossing point. The numerical benchmark finds a single crossing on $[dp - y_1, 3]$ with the general optimiser, and the quantitative model finds exactly one sign change of the buy-minus-rent value along cash at every age and productivity state (Section 6); a general proof is left open.

Proof of Proposition 2. $\hat{b}$ is defined by $D(\hat{b}; \xi) = 0$ for a parameter ξ , with $\partial D/\partial b = 1/c_1^O - 1/c_1^R > 0$ at $\hat{b}$ (proof of Proposition 1). Because the Euler inequalities are strict, a small change in ξ leaves the corner allocation optimal, so the derivative of each value function with respect to ξ is the partial derivative of (22)–(23) at the fixed allocation. By the implicit function theorem $\text{sign}(\partial \hat{b}/\partial \xi) = -\text{sign}(\partial D/\partial \xi)$. The benchmark has no later purchase, so V^R depends on none of $\theta, \kappa_b, \kappa_s, m, \bar{\pi}$; this is the restriction named in the statement. We compute $\partial D/\partial \xi$ for each parameter, using $E_3 \equiv \mathbb{E}[1/W_3^O(y_3)]$ and the bounds implied by (12):

$$\frac{\beta}{c_2^O} < \frac{1}{Rc_1^O}, \quad \beta^2(1 + \nu)E_3 < \frac{\beta}{Rc_2^O} < \frac{1}{R^2c_1^O}. \quad (24)$$

Rent. $\partial V^R/\partial \rho = -1/c_1^R - \beta/c_2^R < 0$ and V^O does not depend on ρ , so $\partial D/\partial \rho > 0$.

Premium and stage-1 income. $\partial D/\partial \chi = 1$. $\partial D/\partial y_1 = 1/c_1^O - 1/c_1^R = \partial D/\partial b$, hence $\partial \hat{b}/\partial y_1 = -1$.

Costs. $\partial V^O/\partial \kappa_b = -p/c_1^O < 0$; $\partial V^O/\partial m = -p/c_1^O - \beta p/c_2^O < 0$; $\partial V^O/\partial \kappa_s = -\beta^2(1 + \nu)pE_3 < 0$; V^R is unaffected.

Price. $\partial V^O/\partial p = -d/c_1^O - \beta(\delta_2 + m)/c_2^O + \beta^2(1 + \nu)e_3E_3$. If $e_3 \leq 0$ all three terms are non-positive and the first is strictly negative. If $e_3 > 0$, use the second Euler inequality in the form $\beta/c_2^O > \beta^2R(1 + \nu)E_3$ on the middle term:

$$\frac{\partial V^O}{\partial p} < -\frac{d}{c_1^O} + \beta^2(1 + \nu)E_3[e_3 - R(\delta_2 + m)], \quad e_3 - R(\delta_2 + m) = (1 - \kappa_s) - (1 - \theta)R^2 - Rm,$$

where the last equality uses $\delta_3 + R\delta_2 = (1 - \theta)R^2$ from (3). If the bracket is non-positive we are done. Otherwise apply (24):

$$\frac{\partial V^O}{\partial p} < -\frac{1}{c_1^O} \left[d - \frac{(1 - \kappa_s) - (1 - \theta)R^2 - Rm}{R^2} \right] = -\frac{1}{c_1^O} \left[1 + \kappa_b + m + \frac{m}{R} - \frac{1 - \kappa_s}{R^2} \right] < 0,$$

because $R \geq 1$ implies $(1 - \kappa_s)/R^2 < 1$. Hence $\partial D/\partial p < 0$.

Down payment. With the loan-to-value constraint binding, $B = (1 - \theta)p$ and the payments are $\delta_2 p$, $\delta_3 p$. Let $\tilde{\delta}_2 \equiv -\partial \delta_2/\partial \theta = R - (1 - a)/(1 + \bar{\pi}) > 0$ and $\tilde{\delta}_3 \equiv -\partial \delta_3/\partial \theta = (1 - a)R/(1 + \bar{\pi})$, so that $\tilde{\delta}_2 + \tilde{\delta}_3/R = R$. Then

$$\frac{\partial V^O}{\partial \theta} = p \left[-\frac{1}{c_1^O} + \frac{\beta \tilde{\delta}_2}{c_2^O} + \beta^2(1 + v)\tilde{\delta}_3 E_3 \right] < \frac{p}{c_1^O} \left[-1 + \frac{\tilde{\delta}_2}{R} + \frac{\tilde{\delta}_3}{R^2} \right] = 0,$$

by (24). A higher down payment is present-value neutral and hurts only through liquidity.

Anticipated inflation. $\partial \delta_2/\partial \bar{\pi} = (1 - \theta)(1 - a)/(1 + \bar{\pi})^2 \equiv x > 0$ and $\partial \delta_3/\partial \bar{\pi} = -Rx$. Hence

$$\frac{\partial V^O}{\partial \bar{\pi}} = xp \left[-\frac{\beta}{c_2^O} + \beta^2 R(1 + v)E_3 \right] < 0$$

by the second inequality in (12); V^R does not depend on $\bar{\pi}$. □

Proof of Proposition 3. (i) The signs of $\partial \delta_2/\partial \bar{\pi}$, $\partial \delta_3/\partial \bar{\pi}$ and the present-value identity are in the proof of Proposition 2; the threshold result is the last item of that proposition. (ii) Let the stage-2 real payment rise by x and the stage-3 payment fall by Rx (a change in $\bar{\pi}$). Replace n_2 by $n_2 - x$, feasible for $x < n_2$: c_2 is unchanged, and $W_3 = y_3 + (1 - \kappa_s)p - D_3 + n_2 R$ changes by $+Rx - Rx = 0$. The original allocation of (c_1, c_2, c_3, b') is therefore still feasible; conversely any allocation feasible after the change is feasible before it with $n_2 + x$, so the feasible sets of consumption plans coincide and the optimum is unchanged. For θ : an increase by x raises the down payment by px and lowers the stage-2 and stage-3 payments by $\tilde{\delta}_2 px$ and $\tilde{\delta}_3 px$. Replace n_1 by $n_1 - px$ and n_2 by $n_2 - \tilde{\delta}_3 px/R$ (feasible for small x): c_1 is unchanged; c_2 changes by $+\tilde{\delta}_2 px - Rpx + \tilde{\delta}_3 px/R = 0$ by $\tilde{\delta}_2 + \tilde{\delta}_3/R = R$; W_3 changes by $+\tilde{\delta}_3 px - \tilde{\delta}_3 px = 0$. The same argument in reverse shows the feasible sets coincide. □

A.3. Leverage

Proof of Proposition 4. (1) $W_3^O = y_3 + (1 - \kappa_s)(1 + g)p - \delta_3 p$, so $\partial W_3^O/\partial [(1 + g)p] = 1 - \kappa_s$; dividing by the stage-1 cash outlay dp gives the multiplier, whose monotonicity is immediate. (2) The owner's housing-related stage-3 wealth is $(1 - \kappa_s)(1 + g)p - \delta_3 p$. The renter-investor pays ρ at stages 1 and 2 instead of dp and $(\delta_2 + m)p$, so saves $dp - \rho$ at stage 1 and $(\delta_2 + m)p - \rho$ at stage 2,

worth $(dp - \rho)R^2 + ((\delta_2 + m)p - \rho)R$ at stage 3. The owner does at least as well iff

$$(1 - \kappa_s)(1 + g) \geq dR^2 + (\delta_2 + m)R + \delta_3 - \frac{\rho}{p}(R^2 + R) = R^2(1 + \kappa_b + m) + mR - \frac{\rho}{p}(R^2 + R),$$

using $dR^2 + \delta_2 R + \delta_3 = (\theta + \kappa_b + m)R^2 + (1 - \theta)R^2$ from (3); θ cancels. (3) Housing equity at resale is $(1 - \kappa_s)(1 - x)p - \delta_3 p \leq 0$ iff $x \geq x^*$. $\square$

A.4. Dynasties and persistence

Proof of Proposition 5. (1) By Lemmas 1 and 2, $b' = vW_3/(1 + v)$ with $W_3^R = y_3$ and $W_3^O = y_3 + e_3 p$; the child owns iff $b' \geq \hat{b}$ (Proposition 1), which gives the two cutoffs. Independence of y_3 across generations makes the tenure sequence Markov. (2) The transition matrix $\begin{pmatrix} 1 - q_{RO} & q_{RO} \\ 1 - q_{OO} & q_{OO} \end{pmatrix}$ has trace $1 - q_{RO} + q_{OO}$ and determinant $q_{OO} - q_{RO}$, hence eigenvalues 1 and λ . For a two-state chain in its stationary distribution, $\mathbb{E}[O' | O = 1] - \mathbb{E}[O' | O = 0] = q_{OO} - q_{RO} = \lambda$, and $\text{Corr}(O, O') = \lambda$ (direct computation with $\Pr(O = 1) = \mu^*$). If μ_t is the owner share of a group, $\mu_{t+1} - \mu^* = \lambda(\mu_t - \mu^*)$. The sign and monotonicity statements follow from (16) with G non-decreasing and strictly increasing where $g > 0$. (3) Standard for an irreducible aperiodic two-state chain: μ^* solves $\mu^* = \mu^* q_{OO} + (1 - \mu^*) q_{RO}$. (4) Immediate from $W_3^O - W_3^R = e_3 p$ and Lemma 1. $\square$

Remark 5 (No income risk: absorbing classes and history dependence). If G is degenerate at $\bar{y}_3$, then $\lambda \in \{0, 1\}$: when $v\bar{y}_3/(1 + v) < \hat{b} \leq v(\bar{y}_3 + e_3 p)/(1 + v)$ the two tenure classes are absorbing, every initial owner share μ_0 compatible with that inequality at $p = S^{-1}(\mu_0)$ is a stationary equilibrium, and the long-run ownership rate is determined by the initial distribution of transfers. This continuum of history-dependent steady states is a knife-edge of the no-risk case: any G with positive density on $(\hat{y}_O, \hat{y}_R)$ gives $\lambda < 1$ and a unique stationary distribution given prices (part 3). Persistence is therefore stated as a half-life, $\ln 2 / (-\ln \lambda)$, not as multiplicity.

A.5. Stationary equilibrium

Proof of Proposition 6. Existence. S^{-1} and S_r^{-1} are continuous on $[0, 1]$; Δ , Λ and $\hat{b}$ are continuous in (p, ρ) where Assumption 3 holds (compositions of continuous functions and of the expectation of a continuous function over a compact support), with $\hat{b} \rightarrow +\infty$ as $\Delta \downarrow 0$ when $dp > \rho$, so the convention $\mu^* = 0$ at $\Delta \leq 0$ makes μ^* continuous; G is continuous. Hence $f : [0, 1] \rightarrow [0, 1]$ is continuous. At $\mu = 0$, $p = 0$, so $dp = 0 \leq \rho$ and every feasible household owns ($\hat{b} = -y_1 < 0$): $f(0) = 1 \geq 0$. At $\mu = 1$, $f(1) \leq 1$. By the intermediate value theorem $f(\mu) - \mu$ has a zero on $[0, 1]$, which is a stationary equilibrium with $p^* = S^{-1}(\mu)$, $\rho^* = S_r^{-1}(1 - \mu)$.

Uniqueness. $\mu^* = q_{RO}/(1 - q_{OO} + q_{RO})$ is increasing in q_{RO} and in q_{OO} ; $q_{RO} = 1 - G(\hat{y}_R)$ is non-increasing in $\hat{y}_R = (1 + v)\hat{b}/v$ and $q_{OO} = 1 - G(\hat{y}_O)$ is non-increasing in $\hat{y}_O = \hat{y}_R - e_3 p$. Along the clearing curve, $dp/d\mu > 0$ and $d\rho/d\mu < 0$, so by Proposition 2

$$\frac{d\hat{b}}{d\mu} = \frac{\partial \hat{b}}{\partial p} \frac{dp}{d\mu} + \frac{\partial \hat{b}}{\partial \rho} \frac{d\rho}{d\mu} \geq \frac{\partial \hat{b}}{\partial p} \frac{dp}{d\mu} > 0, \quad \frac{d\hat{y}_O}{d\mu} = \frac{1+v}{v} \frac{d\hat{b}}{d\mu} - e_3 \frac{dp}{d\mu} \geq \left[\frac{1+v}{v} \frac{\partial \hat{b}}{\partial p} - e_3 \right] \frac{dp}{d\mu} \geq 0$$

under (C). Thus both cutoffs are non-decreasing in μ , f is non-increasing, and strictly decreasing where $g(\hat{y}_R) > 0$. Since f is non-increasing and $\mu \mapsto \mu$ is strictly increasing, $f(\mu) - \mu$ is strictly decreasing on $[0, 1]$ and has at most one zero; existence gives exactly one. $\square$

Proof of Corollary 1. Market clearing forces $\mu^* = \bar{H}$. Under (C), $\mu^*(p, \rho)$ is strictly decreasing in p at fixed ρ (proof of Proposition 6 with $d\rho = 0$), so the clearing price is unique and, by the implicit function theorem, moves in the direction of the demand shift. $\square$

Proof of Proposition 7. With unit demand, every stage-1 and stage-2 household occupies exactly one unit whatever its tenure, so the total demand for units per cohort is 1 and $S(p^*) = 1$ pins p^* . Landlord arbitrage (buy at p , pay $(m + \phi)p$, collect ρ , sell at p next period, discount at R) gives $\rho^* = p^*(1 + m + \phi) - p^*/R$. Nothing else enters either equation. The argument uses unit demand at the first step: if the number of units demanded by a household depended on its wealth or on prices, $S(p^*)$ would equal an aggregate demand that depends on the wealth distribution, and the neutrality would fail (Remark 2). $\square$

A.6. Surprise inflation

Proof of Proposition 8. (1)–(2) Substitute the inflation paths into $D_2 = (i_1 + a)(1 - \theta)p/(1 + \pi_{t+1})$ and $D_3 = (1 + i_2)(1 - a)(1 - \theta)p/((1 + \pi_{t+1})(1 + \pi_{t+2}))$ with $1 + i_1 = R(1 + \bar{\pi})$ and, for the ARM under (E2), $1 + i_2 = R'(1 + \pi_{t+2})$: the $(1 + \pi_{t+2})$ factors cancel and leave $\delta_3 p(R'/R)/(1 + \pi'')$; the FRM keeps $1 + i_2 = R(1 + \bar{\pi})$ and the extra $(1 + \pi'')$ in the denominator remains. The real stage-3 payment of the ARM exceeds $\delta_3 p$ iff $R' > R(1 + \pi'')$, i.e. $1 + r + \phi_r \pi'' > (1 + r)(1 + \pi'')$, i.e. $\phi_r > 1 + r = R$. The cohort aged 3 has a nominal stage-3 payment fixed before the shock, deflated by the realised price level. (3) Real deposit return $R/(1 + \pi'')$ against R ; the wage and rent factors are definitions. (4) The lender's nominal claims are the mortgages net of deposits, so its real revaluation is minus the households' aggregate real revaluation. Condition (18) is the household's net real gain written out, the stage-3 term being the contract-specific erosion of items 1–2; $\Gamma_3 \geq 0$ when $\phi_r \leq R$, and the right-hand side is $O(\pi'')$ with coefficient $y_2(1 - \eta_w) + nR$ against $\delta_2 p + \delta_3 p$ on the left. $\square$

Proof of Proposition 9 and Corollary 2. We prove the corollary; the proposition is the case $n^O = n^R = 0$. Stage-3 owner-parents at $t + 1$ bought at p^* and sell at p_{t+1} , with nominal stage-3 payment

fixed before the shock and predetermined deposits n^O , so their end-of-life wealth is $W_3 = y_3 + E + Rn^O/(1 + \pi^u)$ with $E(p_{t+1}, \pi^u) \equiv (1 - \kappa_s)p_{t+1} - \delta_3 p^*/(1 + \pi^u)$; renter-parents have $W_3 = y_3 + Rn^R/(1 + \pi^u)$. Under Assumption 6 the children's threshold is $\hat{b}(p_{t+1}, \rho_{t+1})$ computed with $p'' = p^*, \rho' = \rho^*$; its partial derivative in p is $-[d/c_1^O + \beta(\delta_2 + m)/c_2^O]/[1/c_1^O - 1/c_1^R] > 0$ (no resale term) and in ρ it is negative as in Proposition 2. Let $Y \equiv (1 + \nu)\hat{b}/\nu$. By Lemma 1 the child of a renter-parent owns iff $y_3 \geq \hat{y}_R \equiv Y - Rn^R/(1 + \pi^u)$ and the child of an owner-parent iff $y_3 \geq \hat{y}_O \equiv Y - E - Rn^O/(1 + \pi^u)$, so the stage-1 owner share is

$$\mu^d(p, \rho, \pi^u) = \mu^*[1 - G(\hat{y}_O)] + (1 - \mu^*)[1 - G(\hat{y}_R)].$$

Let $Q \equiv (\mu^* + \mu_{t+1})/2$, $p = S^{-1}(Q)$, $\rho = S_r^{-1}(1 - Q)$ and $\Psi(Q, \pi^u) \equiv \mu^d(S^{-1}(Q), S_r^{-1}(1 - Q), \pi^u) - (2Q - \mu^*)$; an impact equilibrium is a zero of Ψ . Differentiating the cutoffs at fixed prices, $\partial \hat{y}_O / \partial \pi^u = [-\delta_3 p^* + Rn^O]/(1 + \pi^u)^2$ and $\partial \hat{y}_R / \partial \pi^u = Rn^R/(1 + \pi^u)^2$, so

$$\frac{\partial \Psi}{\partial \pi^u} = \frac{\mu^* g(\hat{y}_O)(\delta_3 p^* - Rn^O) - (1 - \mu^*) g(\hat{y}_R) Rn^R}{(1 + \pi^u)^2} = \frac{\Xi}{(1 + \pi^u)^2},$$

which is strictly positive under Assumption 7 because $g(\hat{y}_O) > 0$. The deposits do not enter the price derivatives:

$$\frac{\partial \mu^d}{\partial p} = -\left[\mu^* g(\hat{y}_O) + (1 - \mu^*) g(\hat{y}_R)\right] \frac{1 + \nu}{\nu} \frac{\partial \hat{b}}{\partial p} + \mu^* g(\hat{y}_O)(1 - \kappa_s) \leq 0$$

under (C'), while $\partial \mu^d / \partial \rho = -[\dots] \frac{1 + \nu}{\nu} \frac{\partial \hat{b}}{\partial \rho} \geq 0$ multiplies $d\rho/dQ < 0$. Hence $\partial \Psi / \partial Q \leq -2 < 0$ and, by the implicit function theorem, $dQ/d\pi^u = -(\partial \Psi / \partial \pi^u) / (\partial \Psi / \partial Q)$ has the sign of Ξ . Items 1–2 of the proposition and the price statements of the corollary follow from $dp/dQ = p/(\varepsilon Q) > 0$ for $\varepsilon \in (0, \infty)$, $d\rho/dQ < 0$ for $\varepsilon_r < \infty$, and $\mu_{t+1} = 2Q - \mu^*$. If $\varepsilon = 0$ the owner stock is fixed, $\mu_{t+1} = \mu^*$, and p_{t+1} solves $\mu^d(p, \rho^*, \pi^u) = \mu^*$ with $\partial \mu^d / \partial p < 0$ (strict because $g(\hat{y}_O) > 0$ and (C') with the $(1 - \mu^*)g(\hat{y}_R)$ term), so $dp/d\pi^u = -(\partial \mu^d / \partial \pi^u) / (\partial \mu^d / \partial p)$ again has the sign of Ξ . Item 3 of the proposition: $d\hat{b} = (\partial \hat{b} / \partial p)dp + (\partial \hat{b} / \partial \rho)d\rho \geq 0$ since both products are non-negative when $\Xi > 0$; hence $\hat{y}_R = Y$ rises and q_{RO} falls. Since $\mu_{t+1} - \mu^* = \mu^*(q_{OO} - q_{OO}^*) + (1 - \mu^*)(q_{RO} - q_{RO}^*) \geq 0$ with the second term non-positive, $q_{OO} \geq q_{OO}^*$, and $\lambda = q_{OO} - q_{RO}$ rises. When $\Xi < 0$ both products are non-positive, so $\hat{b}$ falls; the total derivative of $\hat{y}_R$ is then $\frac{1 + \nu}{\nu} d\hat{b}/d\pi^u + Rn^R/(1 + \pi^u)^2$, the sum of a negative price term and a non-negative deposit term, which gives the unsigned statement of the corollary and its sign when $n^R = 0$. Item 4 is the change in W_3 of stage-3 owners (both terms non-negative), the lender's side of Proposition 8, Lemma 1 applied to the parents' gain minus the higher outlay ddp_{t+1} of the child, and the stage-1 budget of a renter. Item 5: with one stock and unit demand, p_{t+1} solves $S(p) = 1$ and does not depend on π^u . $\square$

B. Numerical algorithm and diagnostics

B.1. Algorithm

The stationary equilibrium is computed as follows. (1) Given prices $(p, \tilde{p}_r)$, the rent from (9), the nominal rate, the estate lottery and the rebate, solve the household problem backward from J to 1: for each age, compute the values of the cash-only branches (rent, buy, refinance, sell-and-rent, move) on the cash-on-hand grid, with the loan chosen among eleven fractions of the binding origination constraint and the house size by comparison of the stored size-specific values, and the value of staying on the full state grid; next-period liquid assets are chosen by exhaustive search on the asset grid followed by a golden-section refinement; the gift at j_g is chosen among twelve fractions of cash; the estate at J is the terminal wealth. (2) Simulate the dynasty panel with common random numbers: 96,000 slots, each holding a parent and a child whose phases are equidistributed, for ten generations of burn-in and six recorded periods, with the gift and the estate transmitted within the slot. (3) Compute excess demands in the two markets, the conditional means of the simulated estates by quintile (the lottery) and the transfer-tax revenue (the rebate). (4) Clear the rental market in $\tilde{p}_r$ by the Illinois regula falsi inside a regula falsi on p for the owner-occupied market, then update the lottery and the rebate with damping 0.7 and repeat until the relative excess demands are below 5×10^{-4} and the lottery changes by less than 10^{-3} . The supply scales $\bar{H}, \bar{H}_r$ are set once, at the baseline, so that $p^* = \tilde{p}_r^* = 1$ (9.41 and 2.90 units).

A transition after an unanticipated shock at $t = 0$ is computed by shooting in the sequence space: guess the log price paths $(\log p_t, \log \tilde{p}_{r,t})_{t < T}$ with $T = 14$, solve the household problem backward from the stationary terminal values with the time-varying prices, rates and contracts, simulate forward from the stationary cross-section with the realised shock, update the stocks by (10), evaluate the excess demands at every date, and update the paths by Broyden steps from a finite-difference Jacobian (step 0.01 in log price, $2(T - 1)$ columns computed in parallel) with periodic refresh; for large shocks a damped tâtonnement pre-phase precedes the first Jacobian. The tolerance is 1.5×10^{-3} on the maximum relative excess demand along the path; runs that stall at the Monte-Carlo noise floor above this value are labelled with their residual in every table. Partial-equilibrium counterparts repeat the forward simulation with prices fixed at the stationary values. The static-expectations variant solves temporary equilibria period by period with stationary continuation values.

B.2. Parameters, grids, tolerances and run times

Grids: liquid assets, 50 points with double-exponential spacing on $[0, 15]$; cash on hand, 120 points on $[0, 22]$; mortgage balance per unit of house, 20 points on $[0, (1 - \theta)p_{\max}]$ with $p_{\max} = 1.6$;

Table B1. Parameters of the baseline (period = 5 years)

Parameter	Value	Unit	Source / status
J	12	periods	spec B.1
j_retire	9	age index	spec B.1
j_gift = j_estate	7	age index	spec B.1
beta (period)	0.8154	-	0.96 annual, illustrative
gamma	2	-	illustrative
alpha	0.2	-	illustrative
chi	0	-	spec B.2 baseline 0
nu	2	-	illustrative
x_shift	1	income units	illustrative
rho_z (period)	0.7738	-	0.95^5, illustrative
sigma_z (period)	0.3043	-	same unconditional variance as annual, illustrative
n_z	5	-	Rouwenhorst
replacement rate	0.5	-	illustrative
sigma_transitory (period, log)	0.03	-	illustrative; unanticipated, breaks cash atoms
h sizes	[1.0, 1.5, 2.25, 3.4]	housing units	spec B.12
s_min	0.3	housing units	illustrative
m (period)	0.05	share of value	0.01 annual, illustrative
kappa_b	0.025	share of price	illustrative
kappa_s	0.05	share of price	illustrative
kappa_f	0.01	share of value	illustrative
delta_H (period)	0.04901	-	0.01 annual, illustrative
N	5	periods	25-year amortisation (literature_review §7)
theta	0.2	-	illustrative (uninsured threshold)
psi	0.39	-	illustrative
stress add-on (period)	0.1218	-	0.02 annual, illustrative
r (period)	0.1041	-	0.02 annual, illustrative
pibar (period)	0.1041	-	0.02 annual (2 % target)
i (period, Fisher)	0.219	-	derived
phi (landlord wedge)	0.05	share of value	illustrative
sigma_seg	1	-	spec B.6
eps	1	-	ILLUSTRATIVE (no verified Canadian estimate)
eps_r	1	-	illustrative
tau_b	0	-	policy experiment
n_a / a_max	50 / 15.0	-	grid
n_b	20	-	grid
n_x	120	-	grid
n_B	11	-	grid
K_estate	5	-	lottery
n_dynasties	96,000	-	simulation
seed	20,261,020	-	simulation

Values in period units after the conversions recorded in src/model.py (annual rates compounded over 5 years; the 5-year AR(1) keeps the unconditional variance of the annual process). “Illustrative” = no verified Canadian estimate in the literature matrix; the supply elasticity in particular has no verified Canadian source (research/literature_review.md, §7). Supply scales $\bar{H}$, $\bar{H}_r$ are set by the normalisation $p^* = p^{r*} = 1$. Source file: results/data/parameters.csv.

four house sizes; five remaining-payment counts; five productivity states; eleven loan fractions; twelve gift fractions; a five-point estate lottery; a consumption floor of 0.01. Units are the average five-year labour income. The baseline stationary equilibrium takes 8 seconds, other stationary equilibria 45–300 seconds, a transition 2.5–8 minutes with the Jacobian on eight processes, and the full pipeline 59.8 minutes on 18 cores: 45 stationary general equilibria (44 within tolerance; the 24,000-dynasty variant stopped at the noise floor), five partial-equilibrium re-solutions, three multiplicity scans, 16 transition experiments (12 within tolerance) and the welfare computations.

B.3. Diagnostics

Table B2. Convergence and numerical diagnostics

Diagnostic	Value
Owner-occupied market, relative excess demand	0.000004
Rental market, relative excess demand	−0.000007
Tolerance (relative excess demand)	2.5e-4
Estate-lottery fixed point: max. change (last round)	0.000009
Bellman residual, owners: mean / 99th pct. / max	0.00067 / 0.0062 / 0.0193
Bellman residual, renters: mean / 99th pct. / max	0.00016 / 0.0008 / 0.0017
Share of households at the forced-sale rule	0.00000
Share of households with a clipped (infeasible interpolated) policy	0.00003
Outer (lottery) rounds / household-block evaluations	1 / 5 (calibration)
Wall-clock time of the baseline (s)	8
Total wall-clock time of make <code>simulate</code> (min)	59.8
Transitions solved (GE converged / total)	12 / 16
<i>Numerical variants (deviation from baseline, in percent of the baseline value)</i>	
grid_coarse	own. -0.83, p^* -0.85, Gini -0.06, gift +0.11, rank–rank +1.55
grid_fine	own. +0.60, p^* -0.11, Gini +0.13, gift +0.10, rank–rank -0.52
tol_tight	own. +0.00, p^* +0.00, Gini +0.00, gift -0.00, rank–rank +0.00
sample_small	own. +0.06, p^* +0.09, Gini -0.06, gift +0.47, rank–rank -0.13
sample_large	own. -0.11, p^* +0.00, Gini +0.13, gift -0.08, rank–rank -0.64
seed_alt	own. -0.10, p^* -0.01, Gini +0.11, gift -0.53, rank–rank +0.09

Bellman residuals are relative differences between the interpolated value function and a re-optimisation on a 400-point fine asset grid at 400 random off-grid states (gift age excluded). Numerical variants: grids (coarse: $n_a = 30, n_b = 12, n_x = 80$; fine: 80, 30, 180), tighter tolerance (10^{-5}), panel of 24,000 and 240,000 dynasties (baseline 96,000), alternative seed. Source files: results/data/baseline/summary.json, results/data/grid_sensitivity.csv, results/data/diagnostics.json.

Table B2 and Figure B1 report the market-clearing residuals, the lottery convergence, the Bellman residuals at off-grid states, the forced-sale and clipped-policy shares and the numerical

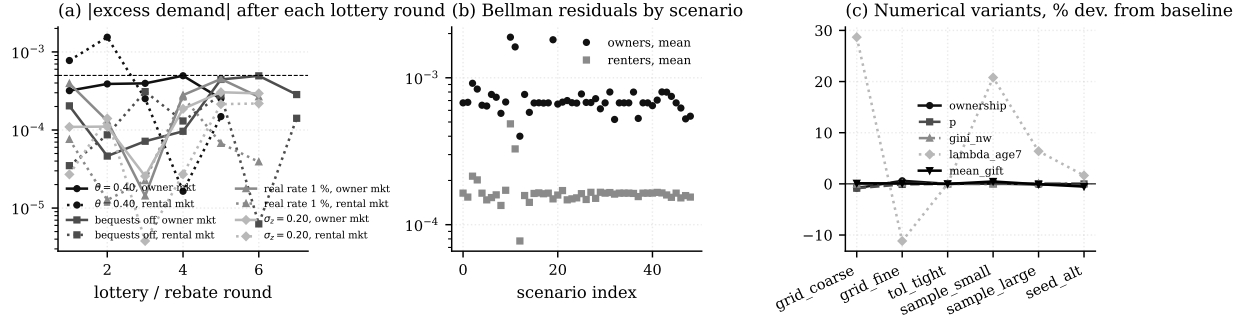


Figure B1. Solver diagnostics: absolute excess demand after each lottery round in the baseline (a), mean Bellman residuals by scenario (b), and percentage deviations of the key statistics from the baseline under the numerical variants (c). Source: results/data/baseline/summary.json, diagnostics.json, grid_sensitivity.csv.

variants. Table B3 lists the economic and numerical robustness runs with their equilibrium statistics and run times. Two events during the computation are recorded because they changed the design. A first full run revealed a genuine discontinuity in the rental excess demand, a jump of 1.5% of rental demand between two adjacent prices in the near-fixed-supply, 5%-down-payment scenario, caused by atoms of the cash distribution: entrants without a gift and liquidity-constrained renters at the same productivity state hold exactly the same cash and switch tenure together. The remedy was the small unanticipated transitory income component of Section 5, after which the excess-demand functions were continuous and the roots bracketed; the budget identities were re-tested with the realised income. And the cold-start shooting of the real-rate shock under a 5% down payment diverged (initial residual 0.17); a warm start from the 20% path and an 80-step damped tâtonnement followed by a fresh Jacobian brought the residual to 8.8×10^{-3} , where Broyden steps were again rejected, so that run is reported as not converged.

C. Notation

Table C1 collects the notation of the quantitative model of Section 3.3; the benchmark of Section 3.2 uses the same symbols with the stage-specific definitions of (2).

Table B3. Robustness: preferences, transaction costs, grids, tolerance, sample size and seed

Variant	Eq.	p^*	Own.	Own. <35	$\hat{a}_1$	Gini	gift	q_{RO}	q_{OO}	rank–rank	min
baseline	GE	1.000	0.631	0.378	0.49	0.542	0.388	0.351	0.538	0.335	0.1
beta_094	GE	0.933	0.581	0.329	0.46	0.576	0.355	0.316	0.496	0.225	2.7
beta_098	GE	1.075	0.681	0.408	0.66	0.498	0.435	0.404	0.597	0.443	3.2
gamma_3	GE	0.995	0.646	0.373	0.47	0.527	0.276	0.375	0.532	0.283	1.8
alpha_025	GE	1.101	0.627	0.359	0.54	0.536	0.412	0.289	0.508	0.362	2.4
kappa_low	GE	1.044	0.622	0.374	0.50	0.543	0.378	0.350	0.524	0.324	1.6
kappa_high	GE	0.962	0.633	0.366	0.52	0.544	0.388	0.335	0.539	0.339	1.4
chi_005	GE	1.006	0.644	0.403	0.48	0.537	0.396	0.355	0.546	0.339	1.0
sigma_low	GE	0.963	0.672	0.318	0.41	0.507	0.304	0.307	0.492	0.321	4.0
sigma_high	GE	1.033	0.602	0.416	0.65	0.565	0.490	0.391	0.570	0.344	1.9
grid_coarse	GE	0.992	0.626	0.369	0.56	0.542	0.388	0.350	0.542	0.341	0.6
grid_fine	GE	0.999	0.635	0.382	0.47	0.543	0.388	0.340	0.535	0.334	1.5
tol_tight	GE	1.000	0.631	0.378	0.49	0.542	0.388	0.351	0.538	0.335	0.2
sample_small	GE [†]	1.001	0.632	0.379	0.50	0.542	0.390	0.340	0.542	0.335	9.9
sample_large	GE	1.000	0.631	0.379	0.49	0.543	0.388	0.349	0.539	0.333	0.6
seed_alt	GE	1.000	0.631	0.378	0.49	0.543	0.386	0.347	0.536	0.336	0.3

Each row is a full re-solution of the stationary general equilibrium (prices, lottery). beta_094/098: annual discount factor 0.94/0.98; gamma_3: $\gamma = 3$; alpha_025: housing share 0.25; kappa_low/high: transaction costs (1%, 2%) / (5%, 8%); chi_005: ownership utility premium 5%; grid_coarse/fine, tol_tight, sample_small/large, seed_alt as in Table B2. Source: results/data/robustness_summary.csv.

Table C1. Notation of the quantitative model

Symbol	Meaning	Units
j	age index, $j = 1, \dots, J = 12$ (5-year periods, 25–84)	–
z	idiosyncratic productivity (Rouwenhorst, $n_z = 5$), frozen in retirement	–
$h \in \mathcal{H}$	owner-occupied house size (four sizes, $h_1 = \underline{h}$)	housing units
s	rented housing services ($s \geq \underline{s}$)	housing units
a	liquid real asset (nominal deposit), $a \geq 0$	goods
b^m	real mortgage balance at the start of the period (B^{nom}/P)	goods
n_m	remaining number of mortgage payments ($N = 5$ at origination)	–
p, p^r, ρ	real house price, rental-unit price, rent per period	goods / unit
P, π, π^u	CPI level, inflation, unexpected inflation	–
R, i, i_q	gross real rate, nominal rate ($1 + i = R(1 + \bar{\pi})$), qualifying rate	–
θ, ψ	minimum down payment, payment-to-income cap	–
$\kappa_b, \kappa_s, \kappa_f, m$	buying, selling, refinancing and maintenance costs	share of value
b^g, b^e	inter-vivos gift (parent’s age $j_g = 7$), estate (parent’s age J)	goods
$v, \underline{x}$	warm-glow weight and luxury shift	–
$\varepsilon, \varepsilon_r, \delta_H$	supply elasticities, depreciation of the stocks	–
$\sigma, \eta_\rho, \eta_w$	segmentation, rent and wage indexation to a surprise	–
$\hat{a}_j(z)$	ownership-entry threshold: smallest liquid wealth at which the age- j renter buys	goods
q_{RO}, q_{OO}, λ	child ownership given parent renter / owner; persistence $\lambda = q_{OO} - q_{RO}$	–
λ^{CE}	consumption-equivalent variation (Section 8.4)	–

Units: “goods” = units of the consumption good; the numeraire of the simulations is the average 5-year labour income of a working-age household ($E[z] = 1$). Source: research/model_specification.md, Part B.